Small Steps Guidance - Percentages & Interest

Year 10

#MathsEveryoneCan

White Rose Maths

	Week 1	Week 2	Week 3	Week 4	Week 5	Week 6	Week 7	Week 8	Week 9	Week 10	Week 11	Week 12
Autumn	**Similarity**						**Developing Algebra**					
	Congruence, similarity and enlargement			Trigonometry			Representing solutions of equations and inequalities			Simultaneous equations		
Spring	**Geometry**						**Proportions and Proportional Change**					
	Angles & bearings		Working with circles		Vectors		Ratios & fractions		Percentages and Interest		Probability	
Summer	**Delving into data**				**Using number**						**Expressions**	
	Collecting, representing and interpreting data				Non-calculator methods		Types of number and sequences		Indices and Roots		Manipulating expressions	

Spring 2: Proportions and Proportional Change

Weeks 1 and 2: Ratios and Fractions

This block builds on KS3 work on ratio and fractions, highlighting similarities and differences and links to other areas of mathematics including both algebra and geometry. The focus is on reasoning and understanding notation to support the solution of increasing complex problems that include information presented in a variety of forms. The bar model is a key tool used to support representing and solving these problems.

National curriculum content covered:

- consolidating subject content from key stage 3:
  - ➢ use ratio notation, including reduction to simplest form
  - ➢ divide a given quantity into two parts in a given part : part or part : whole ratio; express the division of a quantity into two parts as a ratio
  - ➢ relate the language of ratios and the associated calculations to the arithmetic of fractions and to linear functions
  - ➢ use compound units such as speed, unit pricing and density to solve problems
- compare lengths, areas and volumes using ratio notation and/or scale factors; make links to similarity
- apply the concepts of congruence and similarity, including the relationships between lengths, **{areas and volumes}** in similar figures

Weeks 4 and 5: Percentages and Interest

Although percentages are not specifically mentioned in the KS4 national curriculum, they feature heavily in GCSE papers and this block builds on the understanding gained in KS3. Calculator methods are encouraged throughout and are essential for repeated percentage change/growth and decay problems. Use of financial contexts is central to this block, helping students to maintain familiarity with the vocabulary they are unlikely to use outside school.

National curriculum content covered:

- consolidating subject content from key stage 3:
  - ➢ interpret these multiplicatively, express one quantity as a percentage of another, compare two quantities using percentages, and work with percentages greater than 100%
  - ➢ solve problems involving percentage change, including: percentage increase, decrease and original value problems and simple interest in financial mathematics
- set up, solve and interpret the answers in growth and decay problems, including compound interest **{and work with general iterative processes}**

Weeks 5 and 6: Probability

This block also builds on KS3 and provides a good context in which to revisit fraction arithmetic and conversion between fractions, decimals and percentages. Tables and Venn diagrams are revisited and understanding and use of tree diagrams is developed at both tiers, with conditional probability being a key focus for Higher tier students.

National curriculum content covered:

- apply the property that the probabilities of an exhaustive set of mutually exclusive events sum to one
- use a probability model to predict the outcomes of future experiments; understand that empirical unbiased samples tend towards theoretical probability distributions, with increasing sample size
- calculate the probability of independent and dependent combined events, including using tree diagrams and other representations, and know the underlying assumptions
- **{calculate and interpret conditional probabilities through representation using expected frequencies with two-way tables, tree diagrams and Venn diagrams}**

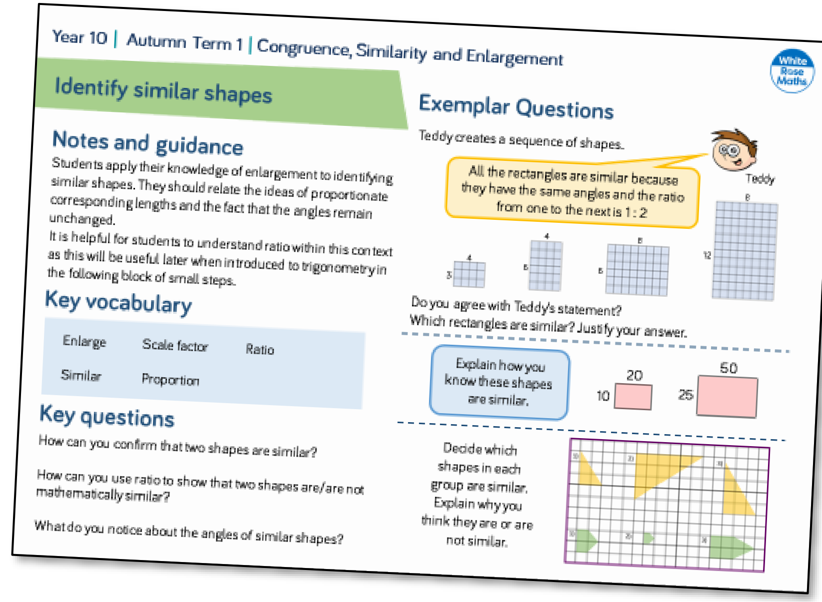
Why Small Steps?

We know that breaking the curriculum down into small manageable steps should help students to understand concepts better. Too often, we have noticed that teachers will try and cover too many concepts at once and this can lead to cognitive overload. We believe it is better to follow a "small steps" approach.

As a result, for each block of content in the scheme of learning we will provide a "small step" breakdown. ***It is not the intention that each small step should last a lesson – some will be a short step within a lesson, some will take longer than a lesson.*** We would encourage teachers to spend the appropriate amount of time on each step for their group, and to teach some of the steps alongside each other if necessary.

What We Provide

- Some ***brief guidance*** notes to help identify key teaching and learning points
- A list of ***key vocabulary*** that we would expect teachers to draw to students' attention when teaching the small step,
- A series of ***key questions*** to incorporate in lessons to aid mathematical thinking.
- A set of questions to help ***exemplify*** the small step concept that needs to be focussed on.



Year 10 | Autumn Term 1 | Congruence, Similarity and Enlargement

Identify similar shapes

Notes and guidance

Students apply their knowledge of enlargement to identifying similar shapes. They should relate the ideas of proportionate corresponding lengths and the fact that the angles remain unchanged.

It is helpful for students to understand ratio within this context as this will be useful later when introduced to trigonometry in the following block of small steps.

Key vocabulary

Enlarge, Scale factor, Ratio, Similar, Proportion

Key questions

How can you confirm that two shapes are similar?

How can you use ratio to show that two shapes are/are not mathematically similar?

What do you notice about the angles of similar shapes?

Exemplar Questions

Teddy creates a sequence of shapes.

Do you agree with Teddy's statement?
Which rectangles are similar? Justify your answer.

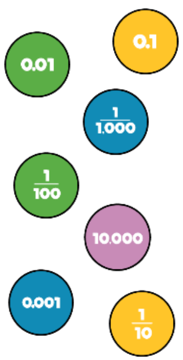
Explain how you know these shapes are similar.

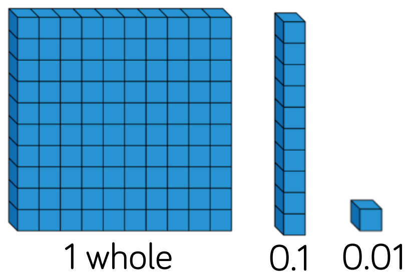
Decide which shapes in each group are similar. Explain why you think they are or are not similar.

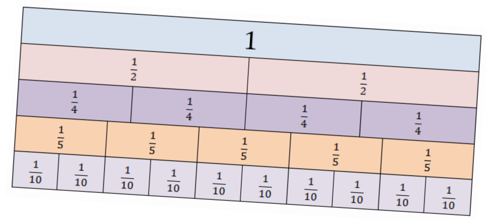
- These include reasoning and problem-solving questions that are fully integrated into the scheme of learning. Depending on the attainment of your students, you many wish to use some or all of these exemplars, which are in approximate order of difficulty. Particularly challenging questions are indicated with the symbol .
- For each block, we also provide ideas for key representations that will be useful for all students.

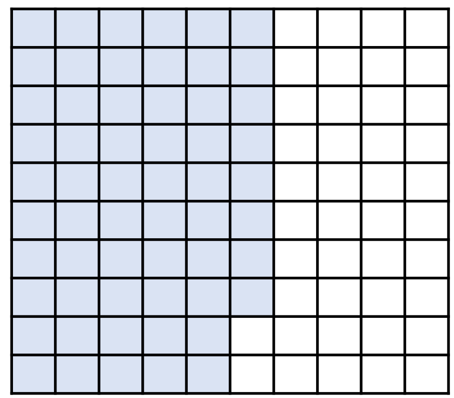
Some of the small steps are in **bold** and labelled with H to indicate this is Higher tier GCSE content. We would encourage teachers to use these with as many students as possible – if you feel your class can access any particular small step, then please include it in your planning. Steps that review content covered at Key Stage 3 are labelled R .

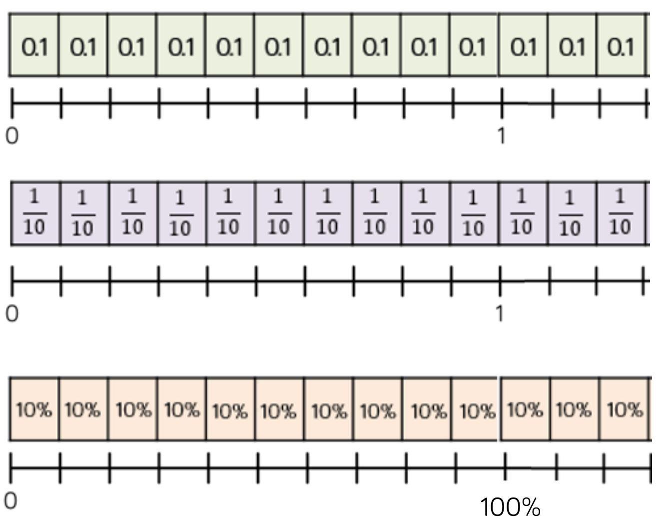
Key Representations

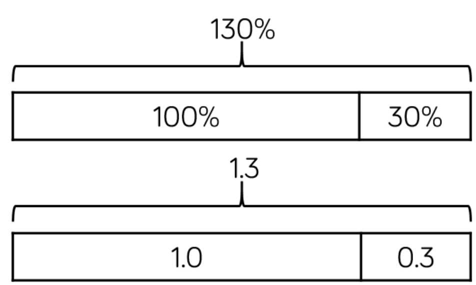


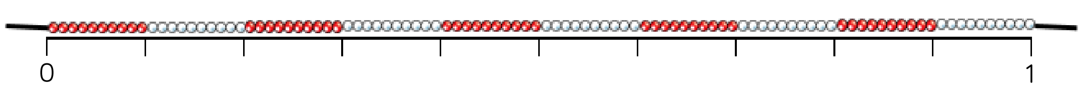












Concrete, pictorial and abstract representations are an important part of developing students' conceptual understanding.

- Number lines are a useful way of assessing whether children understand the size of a F, D or P. Extending the number line above 1 is an option for some students.
- Paper strips can be folded to represent different F, D and P.
- Bar models are particularly useful when comparing F, D, P.
- Bar models are particularly useful to show when an amount has increased above or decreased below 100%
- Number lines can be used to find original amounts for specific given percentage change problems

White Rose Maths

Percentages & Interest

Small Steps

- Convert and compare fractions, decimals and percentages R
- Work out percentages of amounts (with and without a calculator) R
- Increase and decrease by a given percentage R
- Express one number as a percentage of another R
- Calculate simple and compound interest
- Repeated percentage change
- Find the original value after a percentage change R
- Solve problems involving growth and decay

H Denotes Higher Tier GCSE content

R Denotes 'review step' – content should have been covered at KS3

Percentages & Interest

Small Steps

- **Understand iterative processes** H
- Solve problems involving percentages, ratios and fractions

H Denotes Higher Tier GCSE content

R Denotes 'review step' – content should have been covered at KS3

Convert and compare FDP

R

Notes and guidance

Students will be very familiar with these conversions and the amount of time spent on this review step will be dependent on your assessment of your students' needs e.g. you may just include in starter activities or within the teaching of the other steps. It is well worth reminding students how to perform conversions on their calculators as well as through mental and written methods.

Key vocabulary

Fraction	Decimal	Percentage
Equivalent	Convert	

Key questions

Which are the most commonly used percentages? What fractions are they equivalent to?

How can you convert any decimal to a fraction?

Exemplar Questions

Match up the cards that are of equal value.
Write extra cards so that each set contains a fraction, decimal and percentage of equal value.

45% | 0.02 | 0.25 | $\frac{9}{20}$ | $\frac{1}{50}$ | $\frac{1}{4}$

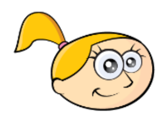
20% | 32% | 0.32

How can $\frac{7}{40}$ be changed to a decimal using a calculator?

Put these cards in order of size, starting with the smallest.

$\frac{1}{3}$ | 0.3 | 28% | 0.33 | 34% | $\frac{13}{40}$ | 0.289

What mistake has Eva made?



$\frac{3}{7} = 3 \div 7 = 0.42 = 42\%$

$0.38 = \frac{38}{100} = \frac{19}{50}$

$0.685 = \frac{685}{1000} = \frac{137}{200}$

Use this method to convert the decimals to fractions, simplifying your answers.

- 0.16
- 0.016
- 0.0016
- 0.925
- 0.08
- 0.78
- 0.204
- 0.555

Find percentages of amounts (R)

Notes and guidance

Students need to be familiar with the use of calculator as well as mental and written methods, linking multipliers to the decimals discussed in the last step. It is also worth looking at multiple methods for a series of calculations to help students decide which methods are most appropriate in a situation. Finding percentages greater than 100% is a useful lead in to reviewing percentage increase in the next step.

Key vocabulary

Fraction	Decimal	Percentage
Equivalent	Convert	Multiplier

Key questions

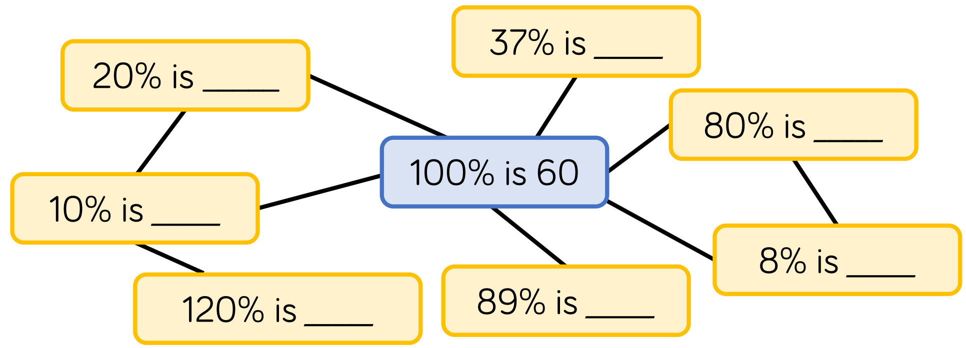
Which percentages are best worked out mentally? Give me an example of a percentage you would work out using a calculator.

How do you work out 10% of a number? How do work out 1% of a number? How are these connected?

Exemplar Questions

Dani is working out 40% of £30 000
Which of these methods will give the correct answer?
Explain why or why not.

$0.4 \times 30\,000$	$30\,000 \div 10 \times 4$	$30\,000 \div 4 \times 10$
$30\,000 \div 5 \times 2$	$30\,000 \div 100 \times 40$	$\frac{40}{100} \times 30\,000$



Complete the spider diagram. Which calculations are best done mentally, and which with a calculator?

Show that 60% of 40 $= \frac{2}{3}$ of 36 $= 0.05 \times 480$

- Is it true that 45% of 80 is the same as 80% of 45?
- Which is greater 34% of 200 or 20% of 350? By how much?
- Find three ways to work out 85% of 90

Increase/decrease by a %age R

Notes and guidance

This review step can also be used to explore different methods and compare their efficacy. Some students get confused when reducing by a given percentage and use the wrong multiplier; the use of estimation is a good strategy here. Confidence with using multipliers is essential for the following steps so it is worth exploring changes of e.g. 3% or 2.7% to avoid over-reliance on mental "build-up" methods.

Key vocabulary

Increase	Decrease	Reduce
Interest	Convert	Multiplier

Key questions

How do you find the multiplier to increase/decrease by __%? How is this different from finding out __% of the number?

What words in a question might mean you need to increase the quantity? What works indicate a decrease?

Exemplar Questions

Esther has £20 000 and she invests £12 000 of this in a bank.
After a year, her investment grows by 5%.
Which of these give the value of her investment after a year?

12 000 × 1.5 | 12 000 + (12 000 ÷ 100 × 5) | 12 000 × 0.05 + 12 000 | 12 000 × 1.05

She buys a car with the remaining £8 000
After a year, the value of the car decreases by 15%.
Which of these give the value of her investment after a year?

8 000 – 0.85 × 8 000 | 8 000 – 0.15 × 8 000 | 0.85 × 8 000 | 8 000 ÷ 100 × 85 | 8 000 × 0.15

In a shop where Nijah works, the cost of a phone is the list price plus VAT at 20%. The list price of a phone is £480
Work out the cost of the phone.
Nijah gets an 18% staff discount on anything she buys from the shop.
How much does it cost Nijah to buy the phone?

Which multiplier increases a number by 3.5%?

× 1.35 | × 3.5 | × 1.035 | × 0.35

Brett earns £36 000 a year.
Calculate his monthly wage after a 3.5 % pay rise.

Express as a percentage

R

Notes and guidance

Students are sometimes challenged when asked to express something as a percentage, rather than the more regular finding of a percentage. A good strategy is to visit this step little and often (perhaps in starters) and to mix the questions students are set rather than treating them discretely. Encouraging students to express as a fraction first and then considering how to convert is also useful.

Key vocabulary

Fraction	Decimal	Percentage
Numerator	Denominator	Express

Key questions

How can I convert any fraction to a percentage using a calculator? If I don't have a calculator, what denominators are useful for converting fractions to percentages?

How can I find a relevant fraction in this question? How can I identify the numerator and denominator?

Exemplar Questions

Convert these test scores into percentages without using a calculator.

 $\frac{30}{40}$  $\frac{32}{40}$  $\frac{36}{40}$  $\frac{28}{40}$

The teacher realises there were only 38 marks on the test, not 40
Use a calculator to work out the new percentage scores, giving your answers to two significant figures.

In a metal bar, the ratio of aluminium to other metals is 2 : 3

- What percentage of the bar is aluminium?

The bar contains 12 kg of aluminium.

- What is the total weight of the bar?

A TV advert for cat food claims that "8 out of 10 cat owners prefer to buy our brand"
In a survey of 72 cat owners, 59 said they preferred to buy the brand.
Use percentages to decide whether you think the claim is true.

	Red	Green	Total
Squares	7	9	16
Circles	18	4	22
Total	25	13	38

The table shows the type and colour of shapes in a game.

- What percentage of the circles are green?
- What percentage of the green shapes are circles?
- What percentage of the shapes are red circles?

Simple and compound interest

Notes and guidance

A useful strategy for helping students to distinguish and remember the difference between simple and compound interest is to compare them alongside each other rather than just looking at them independently. The strategy for compound interest is identical to that of all repeated percentage changes and so will be revisited in many of the upcoming steps.

Key vocabulary

Simple	Compound	Interest
Repeated	Power/Index/Exponent	

Key questions

What is the difference between simple and compound interest? Which one is most common in real life?

What is a quick way of writing (e.g.) $1.07 \times 1.07 \times 1.07$? What buttons do you press on your calculator?

What is a sensible degree of accuracy to use in interest questions? Why?

Exemplar Questions

Alex invests £2 000 at 3% simple interest.

- How much interest will she earn in three years?
- What will her investment be worth after five years?

Fill in the missing numbers.

$6^4 = 6 \times \square$

$1.05 \times 1.05 \times 1.05 = 1.05^{\square}$

$800 \times 1.2 \times 1.2 \times 1.2 \times 1.2 \times 1.2 = 800 \times \square^{\square}$

Annie invests £3 000 at 2% compound interest.
Compare these different ways of calculating the value of her investment after 3 years.

£3 121.20 × 1.02^3
= £3 183.62

£3 000 × 1.02 = £3 060
£3 060 × 1.02 = £3 121.20
£3 121.20 × 1.02 = £3 183.62

How much interest has she earned?
Explain why this is not the same as 6% of £3 000

Find the difference between the interest earned on an £8 000 investment at 4% if the interest is simple or compound.

Tommy is working out formulae for the amount of interest, £I, that £P earns at r% over n years. Are his formulae correct?

Simple Interest

$$I = \frac{Pnr}{100}$$

Compound Interest

$$I = P\left(1 + \frac{r}{100}\right)^n$$

Repeated percentage change

Notes and guidance

This builds on the previous step, generalising the method for compound interest to any repeated percentage change situation, including repeated reduction. Students may not be aware of the term “depreciation”. It is worth considering cases of e.g. an increase of x% followed by a decrease of x% and showing that this does not return to the original value. This is also a good preparation for the next step.

Key vocabulary

Compound	Repeated	Change
Depreciate	Power/Index/Exponent	

Key questions

Why is it that increasing a quantity by (e.g.) 10% twice in a row is not the same as increasing it the quantity by 20%?

What is the overall effect of increasing a number by a percentage and then decreasing it by the same percentage? Why don’t we get back to the original number?

Exemplar Questions

A shop reduces prices by 20% and then by a further 10%

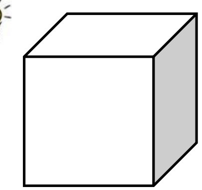
Use calculations to show that Rosie is wrong.

A car costs £15 000 new and loses 18% of its value every year.
Which calculation shows the value of the car in 5 years’ time?

$15\,000 \times 1.18^5$	$15\,000 \times 0.18^5$	$15\,000 \times 0.82^5$

There are 800 000 bacteria in a jar.
The number of bacteria is increasing at a rate of 20% every hour.
How many bacteria will there be in two hours’ time?
How many bacteria will there be in one day’s time?
Give your answer in standard form and to 3 significant figures.

The population of an island is 62 000
It is predicted that the population is decreasing at a rate of 2% a year.
What will the population of the island be in 20 years’ time?
Comment on the accuracy of your answer.



The lengths of the sides of a cube are increased by 20%.
Find the percentage increase in its surface area
Find the percentage increase in its volume

Find the original value

R

Notes and guidance

The exemplar questions show bar models as a way of accessing these problems. Although this will have been covered in KS3, it is worth revisiting as students often make errors such as taking the required percentage off the final value. It is worth looking at multiple methods such as finding 10% or 1% from the given value or using equations of the form “Original × Multiplier = Final Value”.

Key vocabulary

Fraction	Decimal	Percentage
Reverse	Original	Multiplier

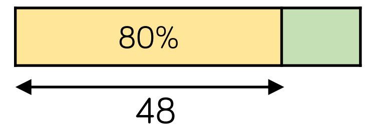
Key questions

If we know (e.g.) 40% of a number, what else can we find?

If an amount is the result of a (e.g.) 20% decrease/increase, what percentage do we know? What other percentages can we find out easily? What percentage is the original value? How can we find this?

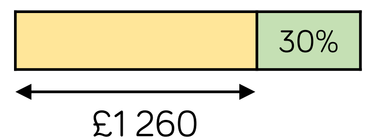
Exemplar Questions

In a test, Whitney answered 80% of the questions correctly.
She answered 48 of the questions correctly.



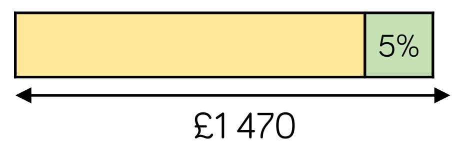
Work out the total number of questions on the test.

In a sale, the price of a TV set is reduced by 30% to £1 260
What percentage of the original price is the sale price?



Work out the original price of the TV.

After a 5% pay rise, Huan earns £1 470 a month.



How much did Huan earn before the pay rise?

Aisha invests £6 000 for 3 years in a saving account.
She gets x% compound interest each year.
Aisha has £6 749.18 at the end of 3 years.
Work out the value of x

Growth and decay

Notes and guidance

This step builds on repeated percentage change, again looking at a variety of contexts. There are no new techniques but students may need to be directed to the links with compound interest and depreciation using the vocabulary of "growth" and "decay". Higher tier students could also consider "working backwards", finding the original value after repeated percentage changes, combining the last two steps.

Key vocabulary

Growth	Decay	Multiplier
Repeated	Compound	

Key questions

If you reduced a number by 50% twice a row, why is the answer not 0?

How can you tell from a question whether the multiplier should be more or less than 1?

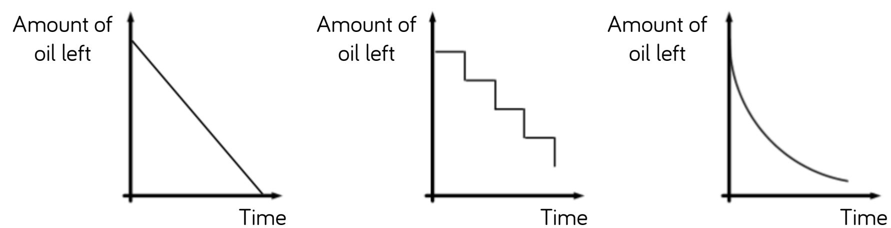
Exemplar Questions

Tom and Dani start work on a salary of £20 000

Tom	***Dani***
4% pay rise every year	£900 pay rise every year

Who will be earning more in five years' time?
Who will be earning more in ten years' time?

There is a hole in an oil tank.
Every hour, 10% of the oil that is left in the tank leaks out.
Which graph shows this?



Describe a situation the other graphs might show.

There are only 500 of a rare species of bird left in the world.
The number of birds is expected to reduce by 10% a year.
If a preservation order is introduced, the number of birds is expected to increase by 5% each year.
Find the difference between the number of birds in 4 years' time depending on whether the preservation order is introduced.

Iterative processes

H

Notes and guidance

Iterative methods for solving equations are covered in Year 11, but if time allows this represents a good opportunity to introduce the notation in the context of repeated change, and also links to the vocabulary of sequences. The differences between linear and geometric sequences and the alternative forms of the rules for the n^{th} term or formalised term-to-term rules can also be explored.

Key vocabulary

Repeat	Iterate	Subscript
$u_n, u_{n+1},$	Term	Geometric

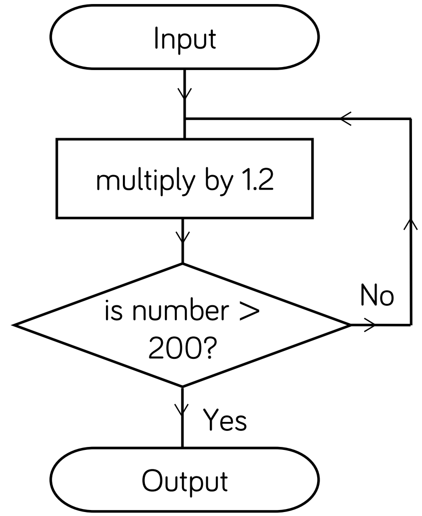
Key questions

What does u_n mean?

Given u_1 and a rule, how many times do I need to iterate in order to find the value of (e.g.) u_5?

Given (e.g.) u_5 and a rule, can I work backwards to find u_1?

Exemplar Questions



- If 100 is input into the flowchart, how many times do you go round the flowchart before getting an answer over 200?
- Investigate the number of iterations for different inputs and different multipliers m if $1 < m < 2$
- Investigate $0 < m < 1$? Will the output ever be 0?

A sequence is given by the rule:

$$u_1 = 10, u_{n+1} = u_n + 6$$

Work out u_2, u_3, u_4 and u_5

- Describe the rule in words.
- Find the rule for the n^{th} term of the sequence.

Repeat for the rule $u_1 = 10, u_{n+1} = 2u_n$

Kim says the rule $u_1 = 1, u_{n+1} = 3u_n$ gives the same sequence as the n^{th} term rule 3^{n-1}

Is she right? Justify your answer.

Problems with FDP and ratio

Notes and guidance

This step provides a nice link with the previous block of learning and can be used to explore examination-style questions that feature a combination of FDP as well as ratio. Bar models and tables are key ways to represent problems to enable students to access the questions which may at first appear overwhelming; teacher modelling of extracting/organising information is extremely helpful.

Key vocabulary

Fraction   Decimal   Percentage

Ratio

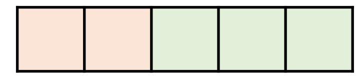
Key questions

What information do you already know? What other information can we work out? How does this help us to solve the given problem?

Is the ratio (e.g.) 2 : 3 the same as the fraction $\frac{2}{3}$? Why or why not?

Exemplar Questions

In a school, the ratio of boys to girls is 2 : 3
What percentage of the students are girls?
10% of the boys wear glasses.
What fraction of the students in the school are boys who wear glasses?



	City	Seaside	Total
Boys			
Girls			
Total			60

A group of 60 children get to choose a school trip.
$\frac{7}{12}$ of the children pick a city trip and the rest pick a seaside trip.
19 boys pick the city trip and 20% of the girls choose the seaside.
Find the ratio of boys to girls in the group.

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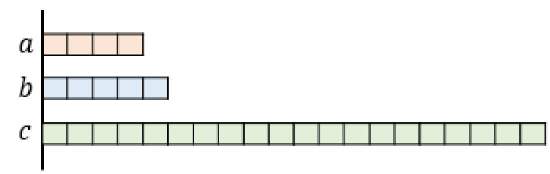
a, b and c are integers.
$a : b = 4 : 5$ and b is 25% of c
Show that a is one-fifth of c



If $a + b + c = 435$, work out the values of a, b and c

Eva, Jack and Rosie share £620
The ratio of Eva's amount to Jack's amount is 8 : 5
The ratio of Jack's amount to Rosie's amount is 2 : 1
How much does Eva get?
What percentage of all the money is Eva's share?