Vectors

Year 10

#MathsEveryoneCan

White Rose Maths

	Week 1	Week 2	Week 3	Week 4	Week 5	Week 6	Week 7	Week 8	Week 9	Week 10	Week 11	Week 12
Autumn	**Similarity**						**Developing Algebra**					
	Congruence, similarity and enlargement			Trigonometry			Representing solutions of equations and inequalities			Simultaneous equations		
Spring	**Geometry**						**Proportions and Proportional Change**					
	Angles & bearings		Working with circles		Vectors		Ratios & fractions		Percentages and Interest		Probability	
Summer	**Delving into data**				**Using number**						**Expressions**	
	Collecting, representing and interpreting data				Non-calculator methods		Types of number and sequences		Indices and Roots		Manipulating expressions	

Spring 1: Geometry

Weeks 1 and 2: Angles and bearings

As well as the formal introduction of bearings, this block provides a great opportunity to revisit other materials and make links across the mathematics curriculum. Accurate drawing and use of scales will be vital, as is the use of parallel line angles rules; all of these have been covered at Key Stage 3. Students will also reinforce their understanding of trigonometry and Pythagoras from earlier this year, applying their skills in another context as well as using mathematics to model real-life situations.

National curriculum content covered:
- interpret and use bearings
- compare lengths…using scale factors
- apply Pythagoras' Theorem and trigonometric ratios to find angles and lengths in right-angled triangles {and, where possible, general triangles} in two dimensional figures
- {know and apply the sine rule and cosine rule to find unknown lengths and angles}
- use mathematical language and properties precisely
- reason deductively in geometry, number and algebra, including using geometrical constructions
- make and use connections between different parts of mathematics to solve problems

Weeks 4 and 5: Working with circles

This block also introduces new content whilst making use of and extending prior learning. The formulae for arc length and sector area are built up from students' understanding of fractions. They are also introduced to the formulae for surface area and volume of spheres and cones; here higher students can enhance their knowledge and skills of working with area and volume ratios.

Higher tier students are also introduced to four of the circle theorems; the remaining theorems will be introduced in Year 11 when these four will be revisited.

National curriculum content covered:
- identify and apply circle definitions and properties, including: centre, radius, chord, diameter, circumference, tangent, arc, sector and segment
- calculate arc lengths, angles and areas of sectors of circles
- calculate surface areas and volumes of spheres, pyramids, cones and composite solids
- apply and prove the standard circle theorems concerning angles, radii, tangents and chords, and use them to prove related results

Weeks 5 and 6: Vectors

Students will have met vectors to describe translations during Key Stage 3 This will be revisited and used as the basis for looking more formally at vectors, discovering the meaning of $-\boldsymbol{a}$ compared to $\boldsymbol{a}$ to make sense of operations such as addition, subtraction and multiplication of vectors. This will connect to exploring 'journeys' within shapes linking the notation $\overrightarrow{AB}$ with $\boldsymbol{b}-\boldsymbol{a}$ etc. Higher tier students will then use this understanding as the basis for developing geometric proof, making links to their knowledge of properties of shape and parallel lines.

National curriculum content covered:
- describe translations as 2D vectors
- apply addition and subtraction of vectors, multiplication of vectors by a scalar, and diagrammatic and column representations of vectors; {use vectors to construct geometric arguments and proofs}.

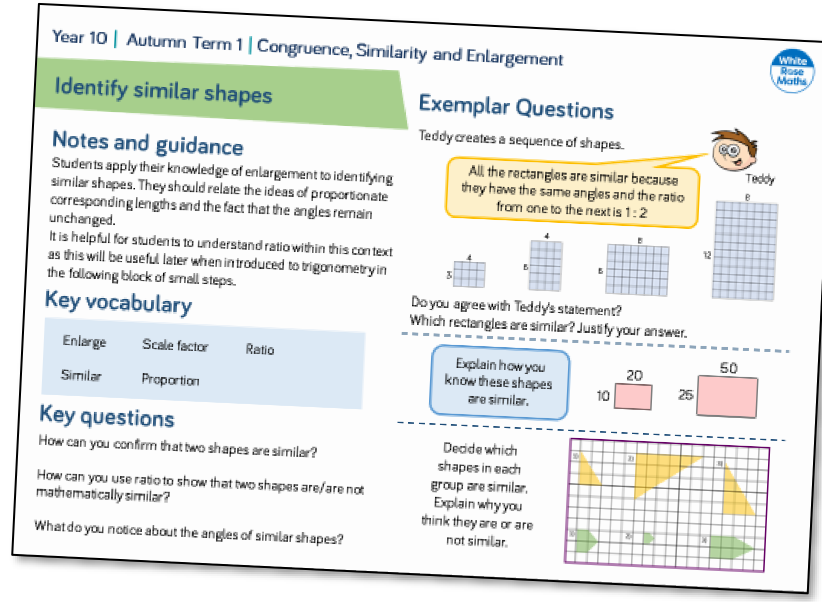
Why Small Steps?

We know that breaking the curriculum down into small manageable steps should help students to understand concepts better. Too often, we have noticed that teachers will try and cover too many concepts at once and this can lead to cognitive overload. We believe it is better to follow a "small steps" approach.

As a result, for each block of content in the scheme of learning we will provide a "small step" breakdown. ***It is not the intention that each small step should last a lesson – some will be a short step within a lesson, some will take longer than a lesson.*** We would encourage teachers to spend the appropriate amount of time on each step for their group, and to teach some of the steps alongside each other if necessary.

What We Provide

- Some ***brief guidance*** notes to help identify key teaching and learning points.
- A list of ***key vocabulary*** that we would expect teachers to draw to students' attention when teaching the small step.
- A series of ***key questions*** to incorporate in lessons to aid mathematical thinking.
- A set of questions to help ***exemplify*** the small step concept that needs to be focussed on.


Year 10 | Autumn Term 1 | Congruence, Similarity and Enlargement

Identify similar shapes

Notes and guidance

Students apply their knowledge of enlargement to identifying similar shapes. They should relate the ideas of proportionate corresponding lengths and the fact that the angles remain unchanged.

It is helpful for students to understand ratio within this context as this will be useful later when introduced to trigonometry in the following block of small steps.

Key vocabulary

Enlarge	Scale factor	Ratio
Similar	Proportion	

Key questions

How can you confirm that two shapes are similar?

How can you use ratio to show that two shapes are/are not mathematically similar?

What do you notice about the angles of similar shapes?

Exemplar Questions

Teddy creates a sequence of shapes.

All the rectangles are similar because they have the same angles and the ratio from one to the next is 1 : 2

Teddy

Do you agree with Teddy's statement?
Which rectangles are similar? Justify your answer.

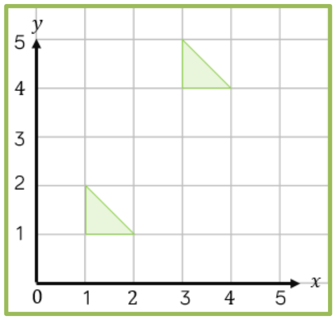
Explain how you know these shapes are similar.

Decide which shapes in each group are similar. Explain why you think they are or are not similar.

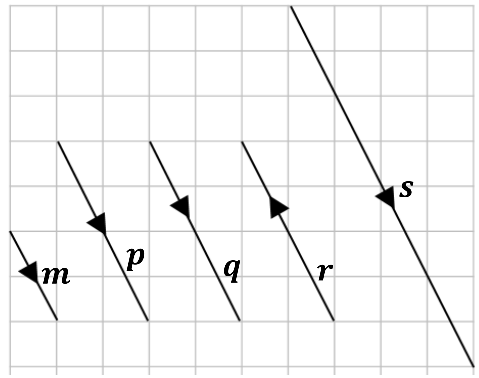
- These include reasoning and problem-solving questions that are fully integrated into the scheme of learning. Depending on the attainment of your students, you many wish to use some or all of these exemplars, which are in approximate order of difficulty. Particularly challenging questions are indicated with the symbol 💡.
- For each block, we also provide ideas for key representations that will be useful for all students.

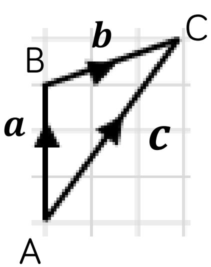
Some of the small steps are in **bold** and labelled with H to indicate this is Higher tier GCSE content. We would encourage teachers to use these with as many students as possible – if you feel your class can access any particular small step, then please include it in your planning. Steps that review content covered at Key Stage 3 are labelled R .

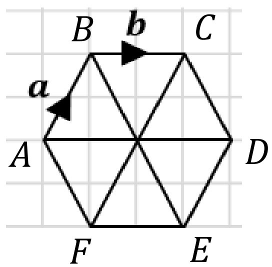
Key Representations



$$\begin{pmatrix} 2 \\ 3 \end{pmatrix}$$







$$\mathbf{f} + \mathbf{g} + \mathbf{h} = \begin{pmatrix} 10 \\ 10 \end{pmatrix}$$

$$\overrightarrow{FC} = 2\overrightarrow{AB}$$

$$\boldsymbol{a} + \boldsymbol{b} = \boldsymbol{c}$$

$$\overrightarrow{AB} + \overrightarrow{BC} = \overrightarrow{AC}$$

$$2\boldsymbol{q} = 2 \times \begin{pmatrix} 2 \\ -4 \end{pmatrix} = \begin{pmatrix} 4 \\ -8 \end{pmatrix}$$

To understand how vectors work, the use of concrete representations and visuals is vital. In initial stages, teachers may want to reinforce the idea of magnitude and displacement by either asking students to physically move objects according to instructions, or to move themselves.

Straws or pipe cleaners can be useful in representing multiplication, addition and subtraction of vectors.

Dynamic geometry packages can be used to visually represent vectors and this can aid production of diagrams from a set of information. Comparing what's the same and what's different about vectors, using diagrams is also helpful.

At all times, concrete and pictorial models should be linked to abstract representations such as $\overrightarrow{OA} = a = \binom{3}{4}$

Vectors

Small Steps

- Understand and represent vectors
- Use and read vector notation
- Draw and understand vectors multiplied by a scalar
- Draw and understand addition of vectors
- Draw and understand addition and subtraction of vectors
- **Explore vector journeys in shapes** H
- **Explore quadrilaterals using vectors** H
- **Understand parallel vectors** H

H Denotes Higher Tier GCSE content

R Denotes 'review step' – content should have been covered at KS3

Vectors

Small Steps

- Explore collinear points using vectors (H)
- Use vectors to construct geometric arguments and proofs (H)

(H) Denotes Higher Tier GCSE content

(R) Denotes 'review step' – content should have been covered at KS3

Understand and represent vectors

Notes and guidance

Students have met vectors previously when translating objects, and so build on from this. A key learning point is that a vector shows both direction and magnitude. It's also important to emphasise the role of the arrow so that students get the idea of starting & end points and hence direction. Comparing vectors with the same magnitude, but different directions is very useful.

Key vocabulary

Column vector	Direction	Scalar
Size	Magnitude	

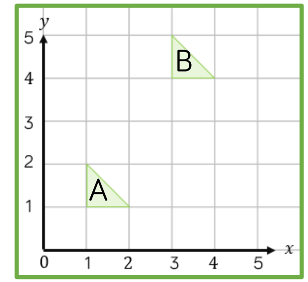
Key questions

What's the same and what's different about a translation and a drawing representing a vector?
What do the numbers in the column vector represent?
How do you know which direction they represent?
What does the arrow show?
How do you know which way to point the arrow?

Exemplar Questions

Write down the column vector that describes the translation from:

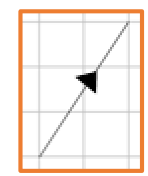
- triangle A to triangle B.
- triangle B to triangle A.



To represent the vector $\begin{pmatrix}2\\3\end{pmatrix}$ on a grid, Dora counts 2 squares to the right and then 3 squares up.

How is this the same/different from translation?

Dora



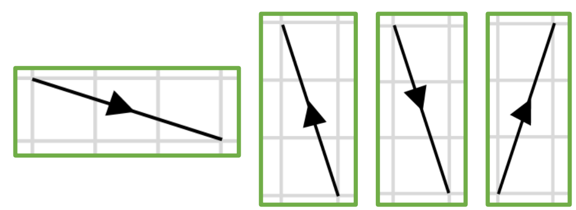
How would this representation differ for vector $\begin{pmatrix}-2\\-3\end{pmatrix}$?

Match up the correct vectors cards. Explain your answers.

3 right and 1 down | 1 left and 3 up | 0 left and 3 up

$\begin{pmatrix}-3\\-1\end{pmatrix}$ $\begin{pmatrix}-1\\3\end{pmatrix}$ $\begin{pmatrix}3\\-1\end{pmatrix}$

$\begin{pmatrix}-1\\-3\end{pmatrix}$ $\begin{pmatrix}3\\1\end{pmatrix}$ $\begin{pmatrix}1\\-3\end{pmatrix}$



Draw a diagram to represent each column vector that doesn't have a match.

Use and read vector notation

Notes and guidance

Students are now familiar with two representations of vectors: column vector and line segment with an arrow. We can now introduce the formal notation for labelling vectors, $\overrightarrow{AB}$and $\boldsymbol{a}$. When handwritten, $\boldsymbol{a}$ is written as a̲. Students develop a deeper understanding of a vector representing movement from one point to another and can start comparing different representations.

Key vocabulary

Column vector	Direction	Magnitude
Size	Arrow	

Key questions

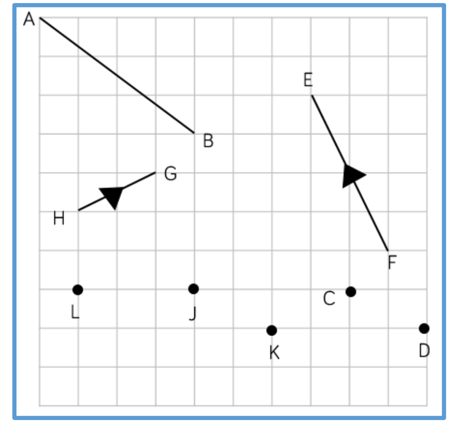
What is the significance of the order of the letters when writing $\overrightarrow{AB}$? What does the arrow tell us?
Do the letters describing a vector always have to be written in alphabetical order?
How do we write a vector using a single lower case letter?

Exemplar Questions

Add an arrow onto the line segment to represent the vector $\overrightarrow{AB}$.
Write $\overrightarrow{AB}$ as a column vector.
How would the column vector $\overrightarrow{BA}$ be different ?

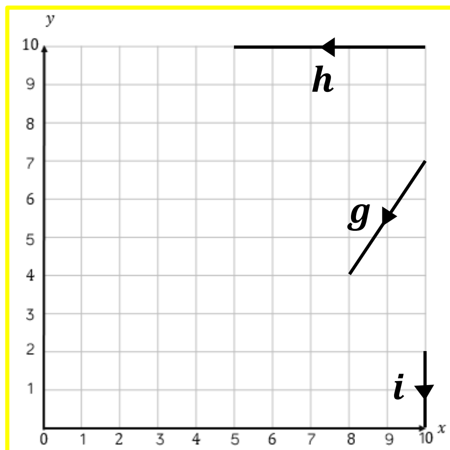
Teddy

Teddy thinks vector $\overrightarrow{EF}$ is shown on the diagram. Is he right? Explain your answer.



Represent the following vectors $\overrightarrow{CD}$ and $\overrightarrow{JK}$on the diagram.
What do you notice about them?

Represent the vector $\overrightarrow{ML} = \binom{0}{3}$ on the diagram.



Write down column vectors to represent $\boldsymbol{g}, \boldsymbol{h}$ and $\boldsymbol{i}$

Copy the axes and plot the points
A (2, 6) and B (4, 3)

Plot a third point, C, to form an isosceles triangle ABC.

Write down the column vectors representing: $\overrightarrow{BA}, \overrightarrow{CB}, \overrightarrow{AC}$

Vectors multiplied by a scalar

Notes and guidance

Students explore vectors that are parallel to each other. They understand that when vectors are parallel, one is a multiple of the other and the multiplier is called a scalar. Students will need support in identifying negative multipliers where vectors are parallel, but are in opposite directions. Looking at diagrammatic and column representations of vectors will help reinforce this.

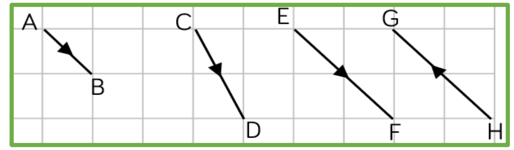
Key vocabulary

Parallel	Scalar	Multiplier
Size	Direction	Equal
Opposite		

Key questions

What's the same and what's different about parallel vectors?
How do we know if one vector is a multiple of another?
How do we know if the multiplier is negative?

Exemplar Questions



Express $\overrightarrow{AB}$ and $\overrightarrow{EF}$ as column vectors. What do you notice?

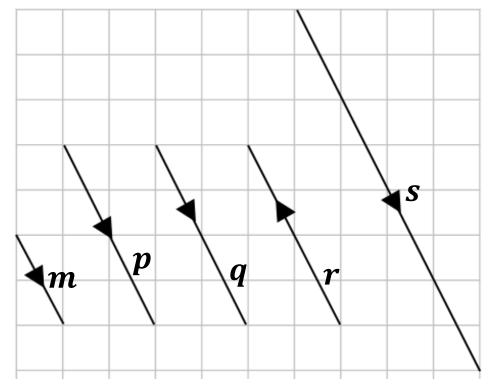
Whitney

Whitney says that $\overrightarrow{CD}$ is double $\overrightarrow{AB}$ as it is twice as long. Explain why Whitney is incorrect.

Draw vector $\overrightarrow{JK}$ so that $\overrightarrow{JK} = 3\overrightarrow{AB}$

Complete the following statement: $\overrightarrow{GH} = \square \times \overrightarrow{HG}$
How do you know the multiplier is a negative number?

In pairs, discuss: What's the same and what's different about the vectors shown in the diagram?
Which of the following statements are true?
(for each statement, explain your answer).



- All of the vectors are parallel
- Vectors $\boldsymbol{p}$, $\boldsymbol{q}$ and $\boldsymbol{r}$ are all equal
- If vectors are parallel, they are always equal
- $\boldsymbol{s} = 2\boldsymbol{r}$
- $2\boldsymbol{q} = 2 \times \begin{pmatrix} 2 \\ -4 \end{pmatrix} = \begin{pmatrix} 4 \\ -8 \end{pmatrix}$
- $\boldsymbol{s}$ is twice as long as $\boldsymbol{p}$ and in the same direction, so $\boldsymbol{s} = 2\boldsymbol{p}$
- If you multiply vector p by −1 this gives vector q
- $\boldsymbol{r} = -\boldsymbol{p}$

Addition of vectors

Notes and guidance

The aim of this small step is for students to become confident in identifying and drawing representations of vector addition. A common misconception is thinking that the resultant vector follows on from the direction of the other vectors. To avoid this, students may need lots of practice in drawing out vector representations of addition. This can be extended to more than two vectors.

Key vocabulary

Direction	Column vector
Addition	Resultant

Key questions

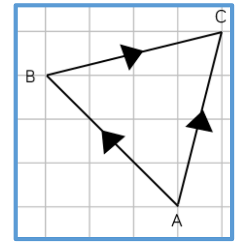
What do we mean by the word 'resultant'?

How do we identify the resultant of two vectors? How does this relate to column vector addition?

If we are adding three vectors together, how can we identify the resultant?

Exemplar Questions

$\overrightarrow{AB} = \begin{pmatrix}-3\\3\end{pmatrix}$ and $\overrightarrow{BC} = \begin{pmatrix}4\\1\end{pmatrix}$

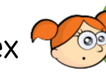


Complete: $\overrightarrow{AB} + \overrightarrow{BC} = \begin{pmatrix}-3\\3\end{pmatrix} + \begin{pmatrix}4\\1\end{pmatrix} = \begin{pmatrix}1\\?\end{pmatrix}$

Compare this to the column vector representing $\overrightarrow{AC}$

Eva and Alex are thinking about how they can find $\overrightarrow{AC}$ using $\overrightarrow{AB}$ and $\overrightarrow{BC}$. Who is right? Why?

Eva  $\overrightarrow{AB} + \overrightarrow{BC} = \overrightarrow{CA}$

Alex  $\overrightarrow{AB} + \overrightarrow{BC} = \overrightarrow{AC}$

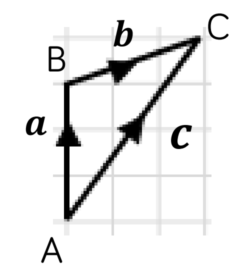
Write down vectors $\boldsymbol{a}$, $\boldsymbol{b}$ and $\boldsymbol{c}$ as column vectors.

Annie works out $\boldsymbol{a} + \boldsymbol{b}$

She writes down $\begin{pmatrix}3\\0\end{pmatrix} + \begin{pmatrix}3\\1\end{pmatrix} = \begin{pmatrix}6\\1\end{pmatrix}$

What mistake has she made?

Calculate, $\boldsymbol{b} + \boldsymbol{a}$ $\quad 2\boldsymbol{b} + \boldsymbol{a}$ $\quad \boldsymbol{b} + \boldsymbol{c}$



Write down possible vectors for $\boldsymbol{f}$ and $\boldsymbol{g}$ and $\boldsymbol{h}$ for each card.

$\mathbf{f} + \mathbf{g} + \mathbf{h} = \begin{pmatrix}10\\10\end{pmatrix}$ $\quad \mathbf{f} + \mathbf{g} + \mathbf{h} = \begin{pmatrix}100\\100\end{pmatrix}$ $\quad \mathbf{f} + \mathbf{g} + \mathbf{h} = \begin{pmatrix}1\\1\end{pmatrix}$

Can you do this so that in all three cases, all the vectors are parallel **or none of the vectors are parallel?**

Draw a diagram to show that, $\overrightarrow{EF} + \overrightarrow{FG} + \overrightarrow{GH} = \overrightarrow{EH}$

Addition & subtraction of vectors

Notes and guidance

This can be introduced by considering both $\boldsymbol{a} + (-\boldsymbol{b})$ and $\boldsymbol{a} - \boldsymbol{b}$ and allowing students to explore equivalence. Students should also be exposed to the pictorial form of this. Adding and subtracting vectors abstractly, including in situations where there are more than two vectors then builds on this. Finally, developing reasoning is key here, so activities such as 'true/false' or 'always, sometimes, never' are useful.

Key vocabulary

Addition	Subtraction	Size
Direction	Multiplying	Scalar

Key questions

What does the arrow on a vector indicate?

What's the relationship between $\boldsymbol{b}$ and $-\boldsymbol{b}$?

How can we identify vector addition on a diagram? Which way around do the arrows go?

Exemplar Questions

$\boldsymbol{a} = \begin{pmatrix} 5 \\ 3 \end{pmatrix}$ $\boldsymbol{b} = \begin{pmatrix} 1 \\ 4 \end{pmatrix}$

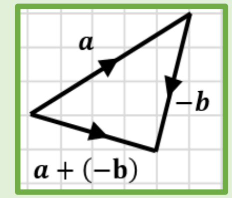
Write down the vector $-\boldsymbol{b}$

Amir is comparing $\boldsymbol{a} + (-\boldsymbol{b})$ with $\boldsymbol{a} - \boldsymbol{b}$

$\boldsymbol{a} + (-\boldsymbol{b}) = \begin{pmatrix} 5 \\ 3 \end{pmatrix} + \begin{pmatrix} -1 \\ -4 \end{pmatrix}$

$= \begin{pmatrix} 4 \\ -1 \end{pmatrix}$

$\boldsymbol{a} - \boldsymbol{b} = \begin{pmatrix} 5 \\ 3 \end{pmatrix} - \begin{pmatrix} 1 \\ 4 \end{pmatrix} = \begin{pmatrix} 4 \\ -1 \end{pmatrix}$



The diagram shows:

$\boldsymbol{a} + (-\boldsymbol{b}) = \begin{pmatrix} 4 \\ -1 \end{pmatrix}$

He concludes: $\boldsymbol{a} + (-\boldsymbol{b}) \equiv \boldsymbol{a} - \boldsymbol{b}$

Is he right? Use the calculations and diagram to justify your answer.

$\boldsymbol{r} = \begin{pmatrix} 2 \\ 5 \end{pmatrix}$ $\boldsymbol{s} = \begin{pmatrix} 1 \\ 6 \end{pmatrix}$ $\mathbf{t} = \begin{pmatrix} -3 \\ 0 \end{pmatrix}$

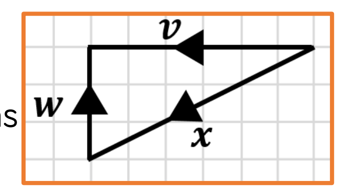
Calculate:

- $\boldsymbol{r} - \boldsymbol{s}$
- $\boldsymbol{s} - \boldsymbol{t}$
- $2\boldsymbol{r} - \boldsymbol{s}$
- $2\boldsymbol{r} - \boldsymbol{s} - \boldsymbol{t}$

Now draw a diagram to represent each calculation.

Write down the column vectors representing $\boldsymbol{v}$, $\boldsymbol{w}$ and $\boldsymbol{x}$.

Calculate the following, writing your answer as a column vector.



$\boldsymbol{v} + \boldsymbol{w}$

$\boldsymbol{v} - \boldsymbol{w}$

Vector journeys in shapes H

Notes and guidance

Students should be encouraged to move around a shape from one vertex to the next, and to write this journey using the notation $\overrightarrow{AB}$ etc. Once they have written the journey in this format, they are then less likely to make mistakes when expressing the same journey using the notation $\boldsymbol{a}$ etc.
Students will require support in identifying when to use a negative in the vector journey.

Key vocabulary

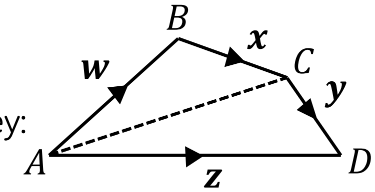
Vector journey	Express
Parallel	Negative

Key questions

How do we know e.g. whether $\overrightarrow{BA}$ is $\boldsymbol{a}$ or $-\boldsymbol{a}$?
If we are writing a vector journey for $\overrightarrow{AD}$, what do you know about the first letter and last letter of each part of the vector journey ($\overrightarrow{AD} = \overrightarrow{AB} + \overrightarrow{BC} + \overrightarrow{CD}$)?
Why is there sometimes more than one way of writing a vector journey?

Exemplar Questions

Annie considers moving along the sides of the shape from vertex A to vertex D.
She writes down the following vector journey:
$\overrightarrow{AD} = \overrightarrow{AB} + \overrightarrow{BC} + \overrightarrow{CD}$



Complete the following to express her journey in terms of $\boldsymbol{w}$, $\boldsymbol{x}$ and $\boldsymbol{y}$:
$\overrightarrow{AD} = \boldsymbol{w} + \square + \boldsymbol{y}$
Complete this vector journey for $\overrightarrow{BC}$.
$$\overrightarrow{BC} = \overrightarrow{BA} + \square + \overrightarrow{DC}$$
$$\overrightarrow{BC} = -\boldsymbol{w} + \square - \boldsymbol{y}$$
Jack and Dora are both writing a vector journey for $\overrightarrow{AC}$.

Dora: $\overrightarrow{AC} = \overrightarrow{AB} + \overrightarrow{BC}$

Jack: $\overrightarrow{AC} = \overrightarrow{AD} + \overrightarrow{DC}$

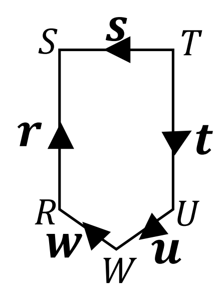
Express Dora's and Jack's vector journeys in terms of $\boldsymbol{w}$, $\boldsymbol{x}$, $\boldsymbol{y}$ and $\boldsymbol{z}$.

Write down two different journeys for vector $\overrightarrow{BC}$.
Now express your journeys in terms of $\boldsymbol{w}$, $\boldsymbol{x}$, $\boldsymbol{y}$ and $\boldsymbol{z}$.

$\overrightarrow{TU} = \boldsymbol{t}$ $\quad \overrightarrow{UW} = \boldsymbol{u}$ $\quad \overrightarrow{WR} = \boldsymbol{w}$ $\quad \overrightarrow{RS} = \boldsymbol{r}$ $\quad \overrightarrow{TS} = \mathrm{s}$

Are these statements true or false?
Correct any false statements.

$\overrightarrow{RT} = \overrightarrow{RS} + \overrightarrow{ST}$	$\overrightarrow{RT} = \overrightarrow{RW} + \overrightarrow{WU} + \overrightarrow{TU}$
$\overrightarrow{RT} = \boldsymbol{r} - \boldsymbol{s}$	$\overrightarrow{RT} = \boldsymbol{w} + \boldsymbol{u} + \boldsymbol{t}$
$\overrightarrow{RT} = \boldsymbol{r} + \boldsymbol{s}$	$\overrightarrow{RT} = -\boldsymbol{w} - \boldsymbol{u} - \boldsymbol{t}$



Quadrilaterals using vectors (H)

Notes and guidance

Students explore quadrilaterals through parallel and non parallel vectors. A common misconception is that parallel vectors are equal; teachers may need to highlight that they have the same direction, but not necessarily the same size. Throughout, students should make generalisations about different quadrilaterals.

Key vocabulary

Parallel	Vector journey	Equal
Opposite	Multiple	Magnitude

Key questions

Which quadrilaterals will have pairs of parallel vectors? How many pairs of parallel vectors will they have?

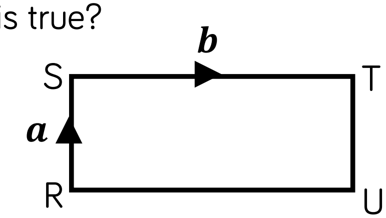
Can a square be described by four equal vectors? Why not? What is the same and what's different about the four vectors that describe a square?

What about the vectors that describe a rectangle?

Exemplar Questions

RSTU is a rectangle. Which statement is true?

$\overrightarrow{TU} = \boldsymbol{a}$ $\overrightarrow{TU} = -\boldsymbol{a}$



Write down $\overrightarrow{UR}$ in terms of $\boldsymbol{b}$.

The coordinate of R is (1, 5) and $\overrightarrow{RS} = \begin{pmatrix} 0 \\ 4 \end{pmatrix}$

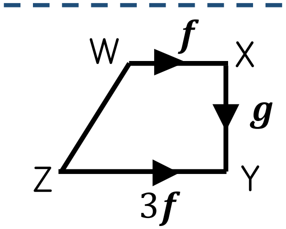
Annie works out the coordinate of S.

Annie

To find S, I need to move up 4 in the y direction, so S is (1, 9)

$\overrightarrow{ST} = \begin{pmatrix} 6 \\ 0 \end{pmatrix}$, now find the coordinates of T and U.

What type of quadrilateral is this?
Explain how you know.
Express $\overrightarrow{WZ}$ in terms of $\boldsymbol{f}$ and $\boldsymbol{g}$.



Draw a rhombus, a parallelogram and a square. Match each statement to the shape type.

There will always be exactly 2 pairs of vectors where one vector in each pair is the multiple of the other.

All vectors have the same magnitude.

There are 2 pairs of parallel vectors, where one pair has different magnitude to the other.

Understand parallel vectors H

Notes and guidance

Students consider parallel vectors represented in different formats e.g. column and pictorial. They appreciate that a vector is only parallel to another if one is a multiple of the other, realising that the multiplier can be negative or fractional. A common misconception can be confusing vectors of equal length with 'equal vectors' – this could be explored by considering e.g. an equilateral triangle.

Key vocabulary

Parallel	Magnitude	Direction
Column	Multiplier	Negative
Fractional		

Key questions

How do we know, without drawing them, whether column vectors are parallel to each other?
How can we identify identical vectors on a diagram?
Are two vectors which are equal in length also identical?

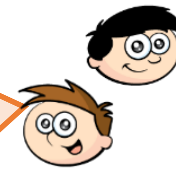
Exemplar Questions

Which five of the following vectors are parallel to each other?
How do you know?

$\begin{pmatrix} 2 \\ 4 \end{pmatrix}$ $\begin{pmatrix} -2 \\ -4 \end{pmatrix}$ $\begin{pmatrix} -2 \\ 4 \end{pmatrix}$ $\begin{pmatrix} 4 \\ -2 \end{pmatrix}$ $\begin{pmatrix} 2 \\ -4 \end{pmatrix}$ $\begin{pmatrix} 8 \\ 4 \end{pmatrix}$ $\begin{pmatrix} 4 \\ 8 \end{pmatrix}$ $\begin{pmatrix} -4 \\ -8 \end{pmatrix}$ $\begin{pmatrix} 3 \\ 6 \end{pmatrix}$

$3\boldsymbol{a}$ $5\boldsymbol{a}$ $6\boldsymbol{a}$

On the green cards, only vectors $3\boldsymbol{a}$ and $6\boldsymbol{a}$ are parallel as 6 is a multiple of 3 and 5 isn't.



All of the vectors on the green cards are parallel as they are all a multiple of a.

Who is correct, Dexter or Teddy? Give a reason for your answer.

$3\boldsymbol{a} + \text{b}$ $5\boldsymbol{a} + \text{b}$ $6\boldsymbol{a} + \text{b}$

If $3\boldsymbol{a}$, $5\boldsymbol{a}$ and $6\boldsymbol{a}$ are parallel, then so are the vectors on the blue cards as $\boldsymbol{b}$ has been added each time.

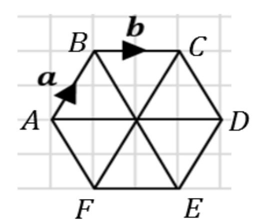
Alex

Explain why is Alex is incorrect.
Which of the following are parallel to $3\boldsymbol{a} + \text{b}$?

$12\boldsymbol{a} + 4\text{b}$ $12\boldsymbol{a} - 4\text{b}$ $\frac{3}{5}\boldsymbol{a} + \frac{1}{5}\text{b}$

ABCDEF is a regular hexagon, so AB ∥ FC.
Write a list of other pairs of parallel lines.
Express the following in terms of $\boldsymbol{a}$ and $\boldsymbol{b}$.

$\overrightarrow{FC}$ $\overrightarrow{ED}$ $\overrightarrow{AD}$ $\overrightarrow{FE}$ $\overrightarrow{AF}$



Collinear points

H

Notes and guidance

In this small step, 'collinear' is likely new language and so the meaning of this will need regular emphasis. Students sometimes struggle to give clear reasons as to why points are collinear (or not) and so may benefit from using stem sentences. It's important to emphasise that a complete reason must be given; sometimes students either state that the lines are parallel or that they share a point.

Key vocabulary

Parallel	Common point
Collinear	Same line

Key questions

Explain why parts of the same line are always parallel to each other.

How can you tell which point both lines pass through?

What does the term 'collinear' mean? When giving reasons for points being collinear, why isn't it enough to just show that two straight lines they create are parallel?

Exemplar Questions

Draw a pair of axes with $0 \leq x \leq 12$ and $0 \leq y \leq 22$

- Plot A (1, 5) and B (4, 10) and express $\overrightarrow{AB}$ as a column vector.
- $\overrightarrow{BC} = \begin{pmatrix} 6 \\ 10 \end{pmatrix}$. Add the point C onto your grid.
- What do you notice about points A, B and C?
- Complete Rosie's reasoning.

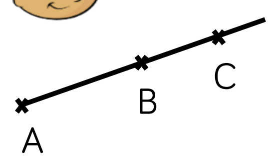
Rosie

$\overrightarrow{BC} = 2\overrightarrow{AB}$ showing that lines BC and AB are ________.
Also, both lines pass through point ___ and so must be part of the ________ line.
If all 3 points are on the same straight line then they are **collinear**.

$\overrightarrow{XY} : \overrightarrow{YZ} = 2 : 3$. If, $\overrightarrow{XY} = 2\mathbf{a} + 4\mathbf{b}$, show that $\overrightarrow{YZ} = 3\mathbf{a} + 6\mathbf{b}$.

Points X, Y and Z are **collinear**. Prove this is correct.

Dora



Dora wants to find C so that points A, B and C are collinear.

- If A (1, 2) and B (7, 4), find $\overrightarrow{AB}$.
- Write down a vector $\overrightarrow{BC}$ which is parallel to $\overrightarrow{AB}$.
- What could the coordinates of point C be?

Geometric argument & proofs H

Notes and guidance

Key command words will need explaining: 'show, justify, prove'. Start by checking confidence in finding vectors for parts of a line segment, given the vector for the whole line segment. Encourage students to draw in extra lines or to extend line segments if necessary. This is an ideal opportunity for goal free questions. Encourage students to write journeys using $\overrightarrow{AB}$ notation first.

Key vocabulary

Show	Justify	Prove
Parallel	Collinear	

Key questions

Can you draw a diagram to represent the situation?
Do you need to add on extra information e.g. points, lines, extend lines?
What do you know? What can you find out?
What do the words 'show, justify and prove' mean?

Exemplar Questions

The point M is $\frac{1}{4}$ of the way along XY.

If $\overrightarrow{XY} = 8\boldsymbol{a} - 4\boldsymbol{b}$, work out $\overrightarrow{XM}$, $\overrightarrow{MY}$ and $\overrightarrow{MX}$ in terms of $\boldsymbol{a}$ and $\boldsymbol{b}$.

If $\overrightarrow{XY} = \boldsymbol{a} + \boldsymbol{b}$, work out $\overrightarrow{XM}$, $\overrightarrow{MY}$ and $\overrightarrow{MX}$ in terms of $\boldsymbol{a}$ and $\boldsymbol{b}$.

True or false?

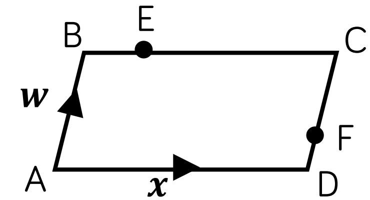
If RS : ST = 1 : 2, then RS is $\frac{1}{2}$ of RT

If $\overrightarrow{RT} = 6\boldsymbol{x} + \boldsymbol{w}$, work out $\overrightarrow{RS}$, $\overrightarrow{ST}$, $\overrightarrow{SR}$ in terms of $\boldsymbol{x}$ and $\boldsymbol{w}$.

If $\overrightarrow{RS} = 6\boldsymbol{x} + \boldsymbol{w}$, work out $\overrightarrow{ST}$, $\overrightarrow{RT}$, $\overrightarrow{TR}$ in terms of $\boldsymbol{x}$ and $\boldsymbol{w}$.

ABCD is a parallelogram.

BE : EC = 1 : 3 and CF = $\frac{3}{4}$ CD.



Express in terms of $\boldsymbol{x}$ and $\boldsymbol{w}$: $\overrightarrow{BD}$ $\overrightarrow{BE}$ $\overrightarrow{AE}$ $\overrightarrow{AF}$

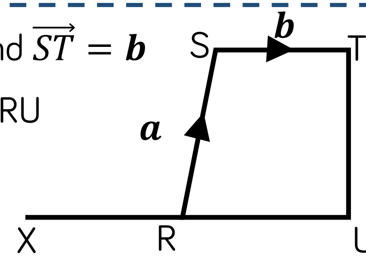
Show that $\overrightarrow{BD}$ is parallel to $\overrightarrow{EF}$

RSTU is a trapezium. $\overrightarrow{RS} = \mathbf{a}$ and $\overrightarrow{ST} = \boldsymbol{b}$

ST is parallel to RU and ST = $\frac{1}{2}$ RU

M is the midpoint of RS

XR : RU = 1 : 2



Not to scale

What do you know? What can you find out? Add this to your diagram.
Prove that points X, M and T are collinear.