Working with Circles

Year 10

#MathsEveryoneCan

White Rose Maths

	Week 1	Week 2	Week 3	Week 4	Week 5	Week 6	Week 7	Week 8	Week 9	Week 10	Week 11	Week 12
Autumn	**Similarity**						**Developing Algebra**					
	Congruence, similarity and enlargement			Trigonometry			Representing solutions of equations and inequalities			Simultaneous equations		
Spring	**Geometry**						**Proportions and Proportional Change**					
	Angles & bearings		Working with circles		Vectors		Ratios & fractions		Percentages and Interest		Probability	
Summer	**Delving into data**				**Using number**						**Expressions**	
	Collecting, representing and interpreting data				Non-calculator methods		Types of number and sequences		Indices and Roots		Manipulating expressions	

White Rose Maths

Spring 1: Geometry

Weeks 1 and 2: Angles and bearings

As well as the formal introduction of bearings, this block provides a great opportunity to revisit other materials and make links across the mathematics curriculum. Accurate drawing and use of scales will be vital, as is the use of parallel line angles rules; all of these have been covered at Key Stage 3. Students will also reinforce their understanding of trigonometry and Pythagoras from earlier this year, applying their skills in another context as well as using mathematics to model real-life situations.

National curriculum content covered:

- interpret and use bearings
- compare lengths…using scale factors
- apply Pythagoras' Theorem and trigonometric ratios to find angles and lengths in right-angled triangles {and, where possible, general triangles} in two dimensional figures
- {know and apply the sine rule and cosine rule to find unknown lengths and angles}
- use mathematical language and properties precisely
- reason deductively in geometry, number and algebra, including using geometrical constructions
- make and use connections between different parts of mathematics to solve problems

Weeks 4 and 5: Working with circles

This block also introduces new content whilst making use of and extending prior learning. The formulae for arc length and sector area are built up from students' understanding of fractions. They are also introduced to the formulae for surface area and volume of spheres and cones; here higher students can enhance their knowledge and skills of working with area and volume ratios.

Higher tier students are also introduced to four of the circle theorems; the remaining theorems will be introduced in Year 11 when these four will be revisited.

National curriculum content covered:

- identify and apply circle definitions and properties, including: centre, radius, chord, diameter, circumference, tangent, arc, sector and segment
- calculate arc lengths, angles and areas of sectors of circles
- calculate surface areas and volumes of spheres, pyramids, cones and composite solids
- apply and prove the standard circle theorems concerning angles, radii, tangents and chords, and use them to prove related results

Weeks 5 and 6: Vectors

Students will have met vectors to describe translations during Key Stage 3 This will be revisited and used as the basis for looking more formally at vectors, discovering the meaning of $-\boldsymbol{a}$ compared to $\boldsymbol{a}$ to make sense of operations such as addition, subtraction and multiplication of vectors. This will connect to exploring 'journeys' within shapes linking the notation $\overrightarrow{AB}$ with $\boldsymbol{b}-\boldsymbol{a}$ etc. Higher tier students will then use this understanding as the basis for developing geometric proof, making links to their knowledge of properties of shape and parallel lines.

National curriculum content covered:

- describe translations as 2D vectors
- apply addition and subtraction of vectors, multiplication of vectors by a scalar, and diagrammatic and column representations of vectors; {use vectors to construct geometric arguments and proofs}.

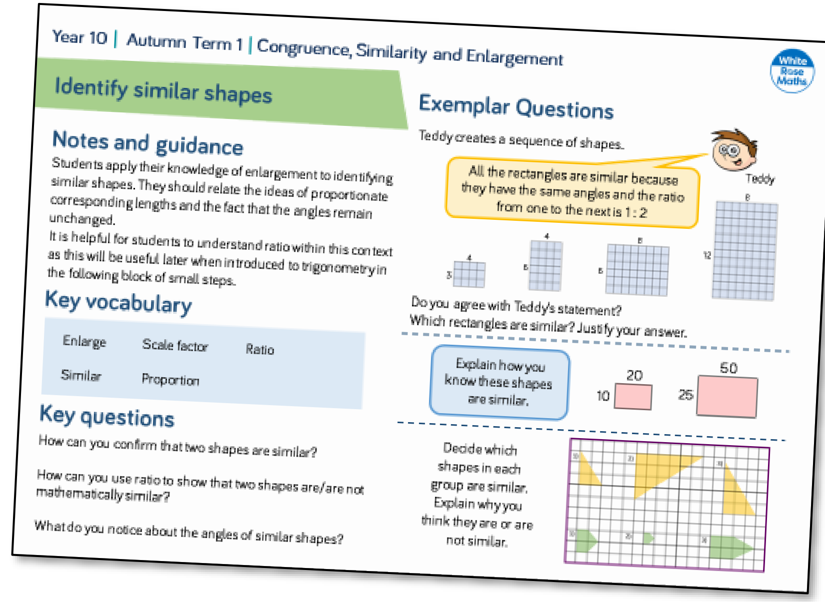
Why Small Steps?

We know that breaking the curriculum down into small manageable steps should help students to understand concepts better. Too often, we have noticed that teachers will try and cover too many concepts at once and this can lead to cognitive overload. We believe it is better to follow a "small steps" approach.

As a result, for each block of content in the scheme of learning we will provide a "small step" breakdown. ***It is not the intention that each small step should last a lesson – some will be a short step within a lesson, some will take longer than a lesson.*** We would encourage teachers to spend the appropriate amount of time on each step for their group, and to teach some of the steps alongside each other if necessary.

What We Provide

- Some ***brief guidance*** notes to help identify key teaching and learning points.
- A list of ***key vocabulary*** that we would expect teachers to draw to students' attention when teaching the small step.
- A series of ***key questions*** to incorporate in lessons to aid mathematical thinking.
- A set of questions to help ***exemplify*** the small step concept that needs to be focussed on.


Year 10 | Autumn Term 1 | Congruence, Similarity and Enlargement

Identify similar shapes

Notes and guidance

Students apply their knowledge of enlargement to identifying similar shapes. They should relate the ideas of proportionate corresponding lengths and the fact that the angles remain unchanged.

It is helpful for students to understand ratio within this context as this will be useful later when introduced to trigonometry in the following block of small steps.

Key vocabulary

Enlarge, Scale factor, Ratio, Similar, Proportion

Key questions

How can you confirm that two shapes are similar?

How can you use ratio to show that two shapes are/are not mathematically similar?

What do you notice about the angles of similar shapes?

Exemplar Questions

Teddy creates a sequence of shapes.

All the rectangles are similar because they have the same angles and the ratio from one to the next is 1 : 2

Do you agree with Teddy's statement? Which rectangles are similar? Justify your answer.

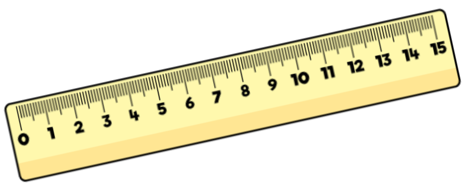
Explain how you know these shapes are similar.

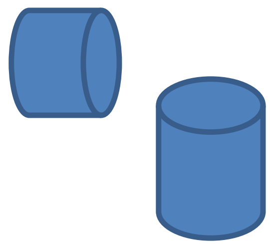
Decide which shapes in each group are similar. Explain why you think they are or are not similar.

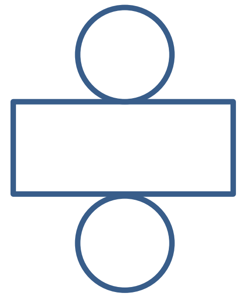
- These include reasoning and problem-solving questions that are fully integrated into the scheme of learning. Depending on the attainment of your students, you many wish to use some or all of these exemplars, which are in approximate order of difficulty. Particularly challenging questions are indicated with the symbol .
- For each block, we also provide ideas for key representations that will be useful for all students.

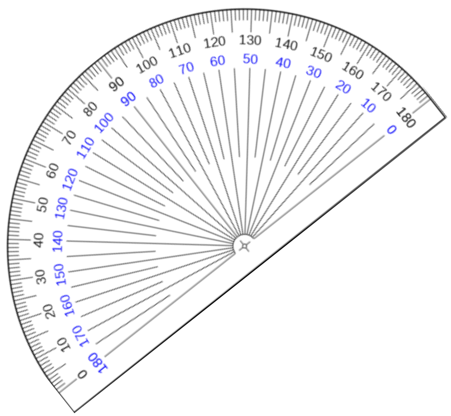
Some of the small steps are in **bold** and labelled with H to indicate this is Higher tier GCSE content. We would encourage teachers to use these with as many students as possible – if you feel your class can access any particular small step, then please include it in your planning. Steps that review content covered at Key Stage 3 are labelled R .

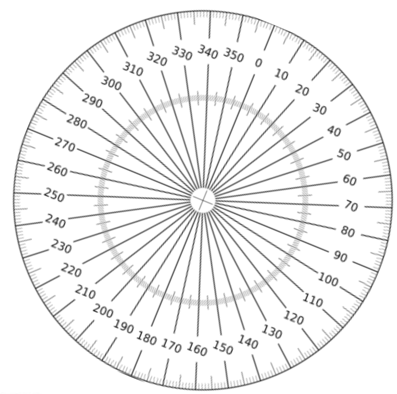
Key Representations

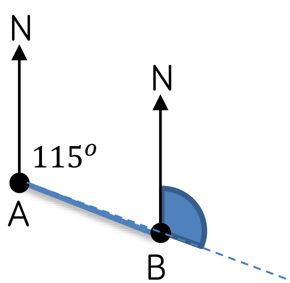


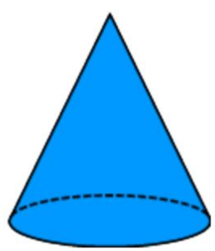












This is a very visual unit of work and students should be encouraged to make accurate diagrams and meaningful sketches throughout.

Using compasses in school or on a trip may be a useful way into bearings. Likewise, making and using actual scale drawings such as architect's drawings is helpful when looking at scale. Straws are always useful for reminding what happens with parallel and intersecting lines.

Creating nets of cylinders and cones is a good way of establishing how to find the formulae for their surface area. Likewise, sand can be used to demonstrate that the volume of a cone is one-third the volume of a cylinder of the same height.

Dynamic geometry is very useful to illustrate the circle theorems.

Working with Circles (1)

Small Steps

- Recognise and label parts of a circle (R)
- Calculate fractional parts of a circle
- Calculate the length of an arc
- Calculate the area of a sector
- **Circle theorem: Angles at the centre and circumference** (H)
- **Circle theorem: Angles in a semicircle** (H)
- **Circle theorem: Angles in the same segment** (H)
- **Circle theorem: Angles in a cyclic quadrilateral** (H)

(H) Denotes Higher Tier GCSE content

(R) Denotes ‘review step’ – content should have been covered at KS3

Working with Circles (2)

Small Steps

- Understand and use the volume of a cylinder and cone
- Understand and use the volume of a sphere
- Understand and use the surface area of a sphere
- Understand and use the surface area of a cylinder and cone
- **Solve area and volume problems involving similar shapes** R H

H Denotes Higher Tier GCSE content

R Denotes 'review step' – content should have been covered at KS3

Parts of a circle

R

Notes and guidance

Students will be familiar with some of the vocabulary associated with circles, but some will be new and will need regular reinforcing over the coming weeks. Showing pupils non-examples that are close in nature to the word in question will help to refine their definitions and understanding. Regular use of geometry packages which make implicit use of the keywords will help memorising.

Key vocabulary

Radius	Diameter	Chord	Centre
Tangent	Arc	Sector	Segment

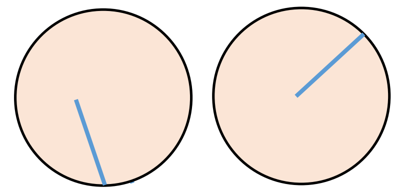
Key questions

Is the diameter of a circle also a chord? Why or why not?
What's the difference between a segment and a sector?
Where do sectors appear in the real world?
What about segments?
Is a tangent/chord a line or a line segment?

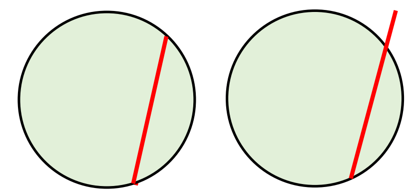
Exemplar Questions

Give reasons for why each diagram is/is not an example of the keyword.

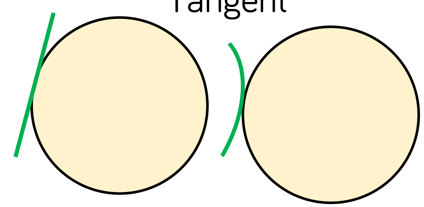
Radius



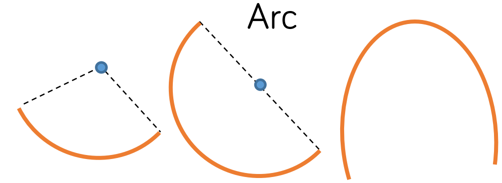
Chord



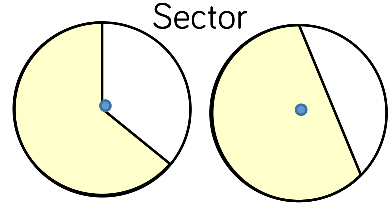
Tangent



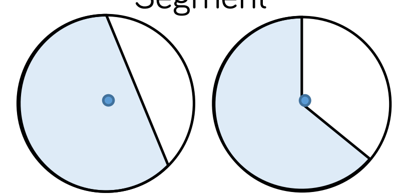
Arc



Sector



Segment



Use a geometry software package to investigate:
What is the maximum number of times that three chords of a circle can intersect? How many times can four chords intersect?
Is it always possible to draw two circles that share the same chord?

Fractional parts of a circle

Notes and guidance

This step reinforces basic fraction work as well as the word 'sector'. If appropriate, the terms major and minor sector could be introduced. Looking at familiar fractions of circles such as quarters and eighths is a useful lead in to the coming steps involving working out arc lengths and areas using formulae of the form $\frac{\theta}{360} \times \cdots$ and shows that complex formulae are not always necessary.

Key vocabulary

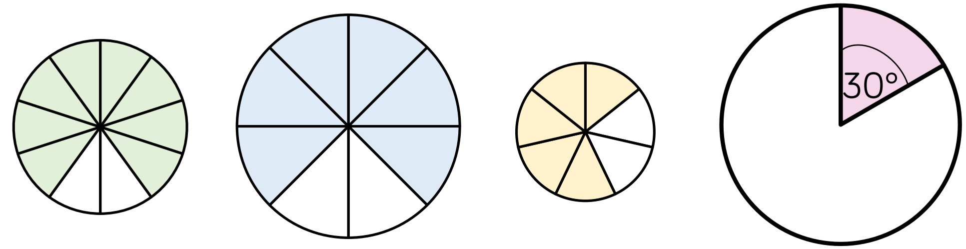
Radius	Diameter	Sector
Arc	Circumference	Area

Key questions

How many degrees are there in a full turn?
What fraction of a circle is represented by … degrees?
What are the formulae for the area and circumference of a circle? How do you remember which is which?

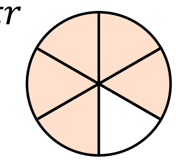
Exemplar Questions

For each circle, write down the fraction that is shaded.



Which is the correct formula for the area of a circle?

$A = \pi r^2$ $\quad A = \pi d$ $\quad A = 2\pi d$ $\quad A = 2\pi r$



The radius of the circle is 6 cm. Find, in terms of π,

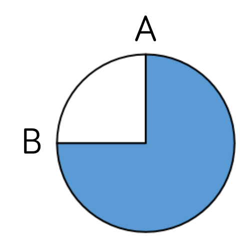
- the area of the circle.
- the area of the shaded region.

A circle has diameter 12 cm. Work out,

- the area of the circle.
- the circumference of the circle.

Tommy says the area and perimeter of a semicircle of diameter 12 cm will be half the answers you have just worked out.
Tommy is only partly right. Explain why.

The circumference of this circle is 20π cm.
What is the distance clockwise from A to B?
What assumption have you made?
Work out the shaded area.



Calculate the length of an arc

Notes and guidance

Students may need to revisit the formula of the circumference of a circle before starting this small step. Building from the previous step, they should realise that the length of an arc is the same fraction of the circumference as the fraction of a full turn given by the related angle. Angles below and above 180 degrees should be explored & both exact and rounded answers should be considered.

Key vocabulary

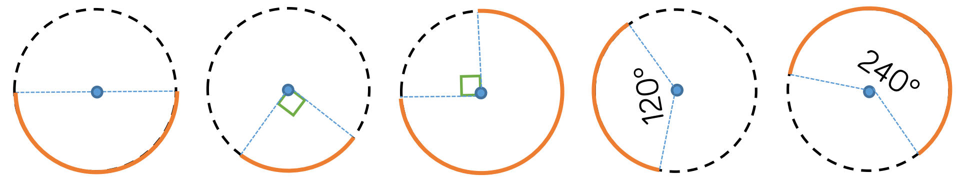
Arc	Circumference	Fraction
Minor/Major	Proportion	Sector

Key questions

What is the formula for the circumference of a circle?
Why is the perimeter of a semicircle not just half the circumference of the circle?
What fraction of a full turn is a 60 degree turn?
How does this help us to find an arc length of a 60 degree sector?

Exemplar Questions

The circumference of this circle is 24 cm. Find the arc length in each case.



Find the arc lengths if the radius of the circle were 24 cm.

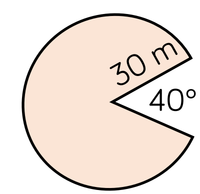
Jack thinks he can find the arc length of any circle with a radius of r and angle of θ

$\frac{\theta}{360} \times$ circumference

Is Jack correct?

Explain each term of the following calculation to find the perimeter of this shape.

$$\frac{320}{360} \times 2\pi \times 30 + 30 + 30$$



Calculate the answer to 1 decimal place.

A circle has radius 10 m. Find, in terms of π, the arc length if,

- the angle at the centre is $36°$
- the angle at the centre is $270°$

The length of an arc subtended by an angle of 120° is 8π cm.
Work out the radius of the circle.

Calculate the area of a sector

Notes and guidance

Students need to be familiar with the formula for the area of a circle. Links should be made with the previous step, establishing that the proportion of a full turn taken up by the sector is identical to its proportion of the area of the circle, leading to the formula $\frac{\theta}{360} \times \pi r^2$

Again, looking at examples both in terms of π and in decimal form is useful, as is working backwards to find θ, r or d.

Key vocabulary

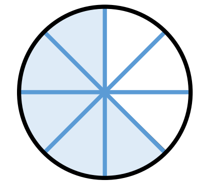
Arc	Area	Fraction
Subtend	Proportion	Sector

Key questions

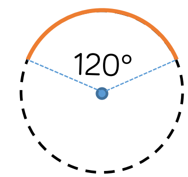
What is the formula for the area of a circle?
If the angle formed by a sector is e.g. 80 degrees, what fraction of the circle is the sector?
How does this help us to find the area of the sector?
Can the angle in the sector be a reflex angle?

Exemplar Questions

The circle is split into equal parts.
The area of the circle is 24π cm^2.
What is the area of the blue sector?
What is the area of the white sector?



The outline for a company logo is the sector of a circle subtended by an angle of 120° as shown in the diagram.

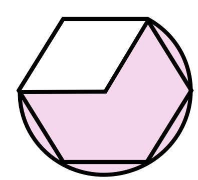


- What is the area of the logo if the radius of the circle is 10 cm?
- What is the area of the logo if the diameter of the circle is 10 cm?

Complete the table.

Radius	Diameter	Angle at centre	Area of sector	Length of arc
5 cm		100°		
	1.2 m	13°		
6 cm			3π cm^2	

Part of a circle is drawn from the centre of a regular hexagon as shown.
The sides of the hexagon are 60 cm.
Work out the shaded area.



Angles: centre, circumference H

Notes and guidance

This first circle theorem needs to be clearly understood as it is the basis of many of the other theorems that follow.
It is useful to include cases with chords/angles in many orientations, not just with the angle at the circumference at 'the top' of the circle. Students need to prove the circle theorems; the use of isosceles triangles in this case is also crucial to many other circle theorem problems.

Key vocabulary

Centre	Circumference
Angle	Isosceles

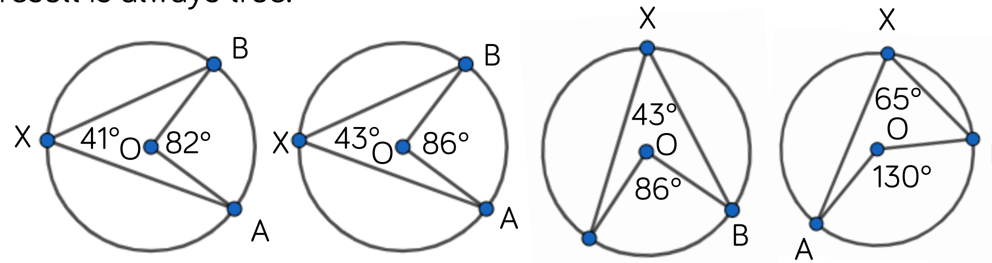
Key questions

How do you identify the 'angle at the centre' and the 'angle at the circumference'?

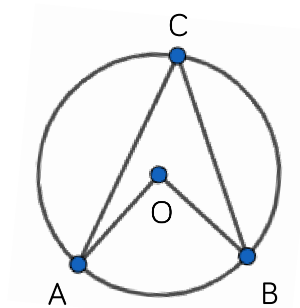
Why do we need to be careful when the angle at the centre is a reflex angle?

Exemplar Questions

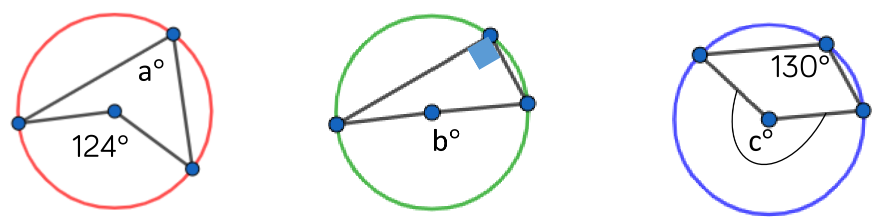
Compare angles AOB and AXB in the diagrams below.
What to you notice? Use dynamic geometry software to see if your result is always true.



Copy the diagram.
Add the radius OC to the diagram.
Write down the names of the isosceles triangles that are formed.
Which angles are equal?
Use these angles to prove that $\angle AOB = 2\angle ACB$



Work out the unknown angles.



What is the same and what is different about your calculations?

Angles in a semicircle

H

Notes and guidance

As well as the proof indicated here, this theorem can be easily proved as a special case of the previous step when the angle at the centre is 180°. Encourage the students to explore near examples of this rule, which could be constructed with geometry software packages. The use of the (unmarked) right angle to revisit Pythagoras' theorem or trigonometry could also be explored.

Key vocabulary

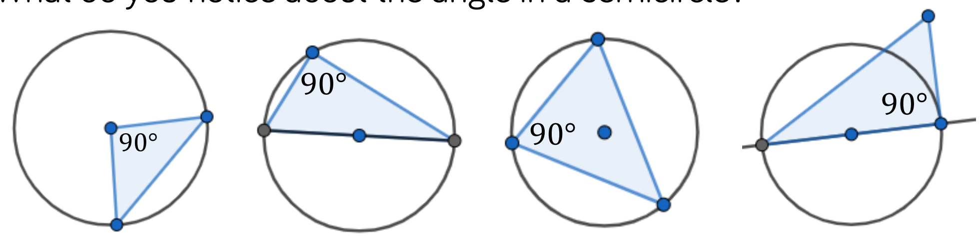
Semicircle	Diameter	Centre
Right angle	Circumference	Pythagoras

Key questions

What is the 'angle at the centre' of a circle if we have a diameter?
What does this mean about the 'angle at the circumference'?
What other rules do we know about right-angled triangles?

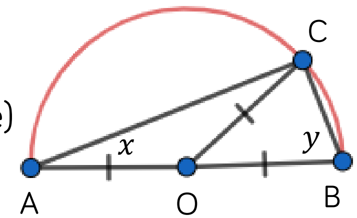
Exemplar Questions

Use dynamic geometry to investigate whether these diagrams are possible, impossible or always true?
What do you notice about the angle in a semicircle?



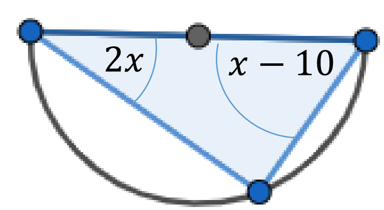
The proof that the angle in a semicircle is always $90°$ is started below. Follow the instructions to help complete the proof.

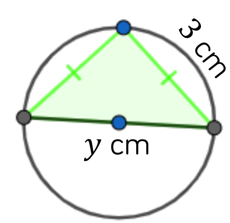
AO = OC (both radii of the same circle)
So $\angle ACO = x$ (Angles in an isosceles triangle)



- Find an expression for $\angle OCB$
- Find the angle sum of triangle ACB in terms of x and y
- Equate this to 180° and find the value of $x + y$

Work out x and y in the semicircles shown.





Angles in the same segment

H

Notes and guidance

This theorem is most commonly proved by showing that angles at the circumference from the same chord/arc share a common angle at the centre of the circle. Students may need help to reinforce the language of ‘segment’, which can often be confused with ‘sector’, and ‘subtend’.
Varying the diagram is again useful so that students do not only look for the common ‘bow-tie’ shape.

Key vocabulary

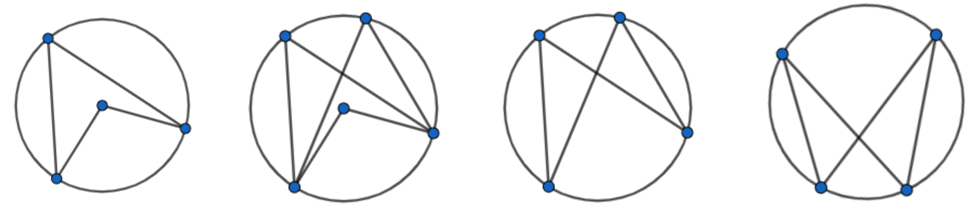
Segment	Chord
Subtend	Arc

Key questions

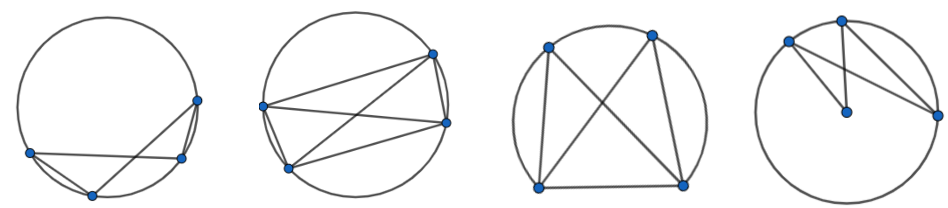
What is the difference between a sector of a circle and a segment of a circle?
How many ‘angles at the circumference’ can be drawn from a single chord? Will they all be equal in size?
Why or why not?

Exemplar Questions

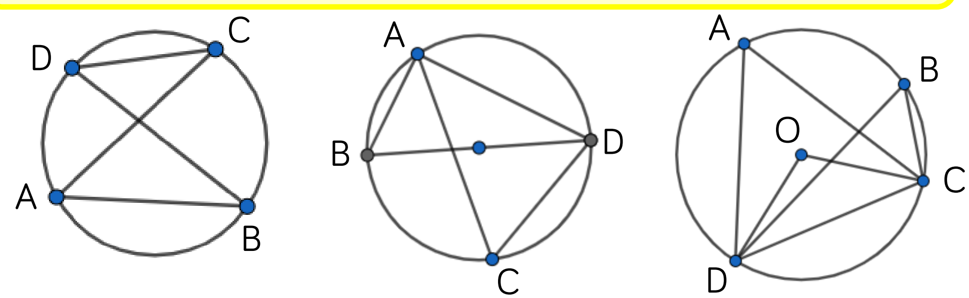
What has changed or stayed the same from diagram to diagram? Colour in angles that are equal in size.
How do you know they are equal?



Which of the diagrams below show that angles in the same segment are the same and which do not? How do you know?



In each of the circles, ∠ABD is 65°. Are any other of the angles 65°? What other angles can you find?



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Angles in a cyclic quadrilateral H

Notes and guidance

This small step is an opportunity to recap students' knowledge of different types of quadrilaterals, their properties and the sum of their interior angles. The fact that opposite angles add to 180° could be explored using a geometry software package, and can be proved again by using angles at the centre and circumference. At this stage students should also explore problems that require the use of more than one circle theorem.

Key vocabulary

Quadrilateral	Cyclic	Circumference
Vertices	Opposite	

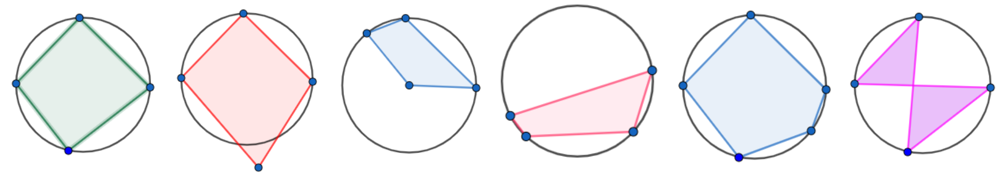
Key questions

What's the difference between opposite angles and adjacent angles? How can we identify the opposite angles of a cyclic quadrilateral?

What does cyclic mean? Are all quadrilaterals cyclic? How do you know?

Exemplar Questions

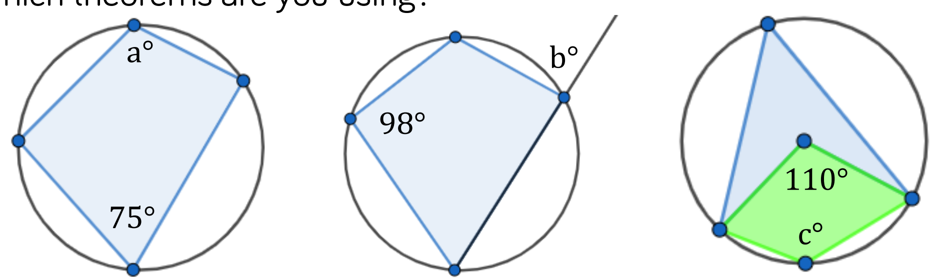
Which of the following diagrams are cyclic quadrilaterals? Explain why or why not.



Are the following statements true or false?

- Opposite angles of a cyclic quadrilateral add up to 180°
- Adjacent angles of a cyclic quadrilateral add up to 180°
- All rectangles are cyclic quadrilaterals.
- It is impossible for a parallelogram to be cyclic.
- It is impossible for a kite to be cyclic.
- A trapezium may or may not be cyclic.

Work out the lettered angles in the diagrams below.
Which other angles can you work out?
Which theorems are you using?



Volume of a cylinder and cone

Notes and guidance

It may be useful to revise the volume of a prism before moving on to the volume of a cylinder, which will have been covered in Year 9. Explore the similarities and differences between the two shapes, using physical demonstrations if possible. You could also point out that a cone is a type of pyramid with a circular base. Students do not need to learn these formulae, but should be fluent in their use.

Key vocabulary

Cylinder	Cone	In terms of π
Perpendicular height	Base	Frustum

Key questions

How do you enter the calculation for the volume of a cylinder/cone into your calculator?

What does the instruction 'leave your answers in terms of π' mean?

How can Pythagoras' theorem help us to work out the perpendicular height of a cone?

Exemplar Questions

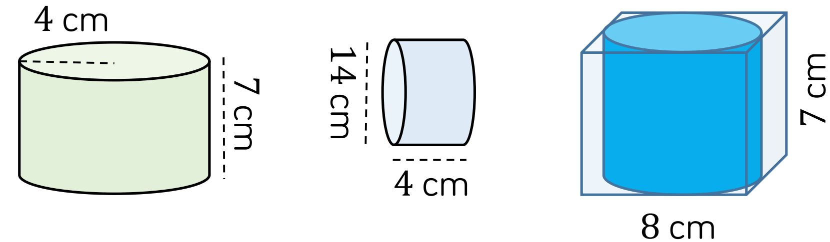
Compare the formulae for the volumes of cones and cylinders.

$$V_{cylinder} = \pi r^2 h \qquad V_{cone} = \frac{1}{3}\pi r^2 h$$

If a cone and a cylinder have the same height and radius, how many times can you empty the full cone into the cylinder?

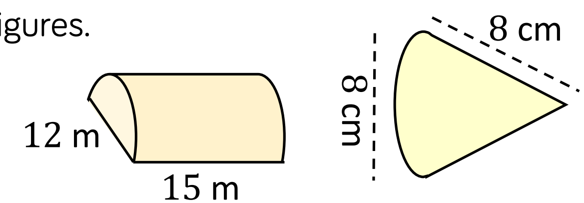
Investigate using sand.

Which cylinder has a volume of $\pi \times 7^2 \times 4$ cm^3?



Work out the volumes of all three cylinders in terms of π and put the cylinders in ascending order of volume.

Find the volume of the shapes, giving your answers to 3 significant figures.



Hint:
Use Pythagoras' theorem to find the height of the cone.

A cone has diameter 12 cm and height 20 cm.
A smaller cone of height 10 cm is cut off the top of the cone.
Work out the volume of the remaining shape.

Volume of a sphere

Notes and guidance

Students need to be careful using this formula as both the fraction and the cubing can cause problems.
The use of a calculator could be modelled and compared with non-calculator methods. Having now looked at three shapes, students could explore the total volume of shapes made by combining these, and also look at parts of the shapes e.g. hemispheres.

Key vocabulary

Sphere	Radius	Diameter
Hemisphere	Centre	

Key questions

How many lengths do you need to know to be able to find the volume of a sphere?

How does the volume of a hemisphere compare to the volume of a sphere?

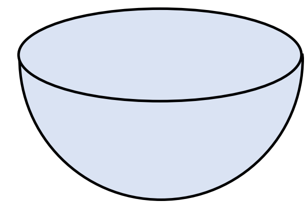
Exemplar Questions

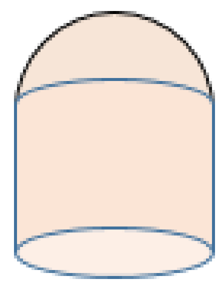
Use the formula $V = \frac{4}{3}\pi r^3$ to find the volume of,

- a sphere of radius 6 cm.
- a sphere of diameter 6 cm.

Give your answers in terms of π.

The volume of a sphere is given by $V = \frac{4}{3}\pi r^3$
The diagram shows a hemisphere of radius 10 cm. Find its volume to 3 significant figures.





The diagram shows a hemisphere that fits exactly on top of a cylinder.
The height of the cylinder is 12 cm.
The height of the whole shape is 18 cm.
What is the radius of the sphere?
Work out the volume of the shape.

A metal cuboid measures 20 cm by 10 cm by 5 cm.
The cuboid is melted and recast into spheres of radius 3 cm.
How many spheres can be made?
Jack thinks that you can make exactly twice as many spheres of radius 1.5 cm from the same block.
Show working to show that Jack is wrong.

Surface area of a sphere

Notes and guidance

This is another given formula, and it would be useful to look at this in conjunction with either the next or previous step so that students experience making the right choice of formula to use. Again, both exact and rounded answers should be considered. You may wish to investigate and compare the structure of area and volume formula, even though dimensional analysis is not required.

Key vocabulary

Surface Area	Curved Surface	Sphere
In terms of π	Radius	Diameter

Key questions

How do you enter the calculation for the surface area of a sphere into your calculator?

How does the surface area of a sphere compare to the area of a circle?

Exemplar Questions

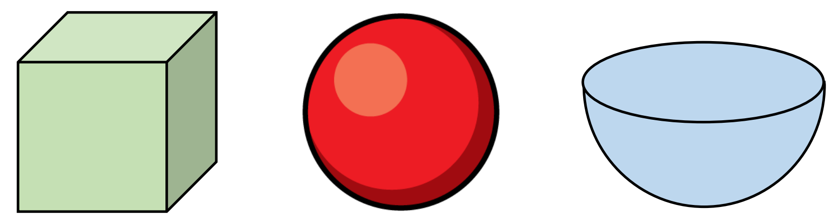
Use the formula $A = 4\pi r^2$ to find the surface area of a sphere if,

- it has radius 10 cm.
- it has diameter 10 cm.

Give your answers in terms of π.

The radius of the small ball is 2 cm. The radius of the large ball is 6 cm.

Alex thinks that the surface area of the large ball will be three times that of the small ball. Use calculations to show that Alex is wrong.



The cube has side length 10 cm.

The sphere has radius 8 cm.

The hemisphere has radius 12 cm.

Put the shapes in order of size given by,

- their surface areas.
- their volumes.

Surface area of cylinder & cone

Notes and guidance

Students should be able to deduce the surface area of a cylinder by considering its net, whilst the formula for the curved surface area of a cone will be given. Pythagoras' theorem may be needed to calculate the slant height or perpendicular height. Allowing students to see the links between the areas by making or deconstructing cylinders and cones is highly effective.

Key vocabulary

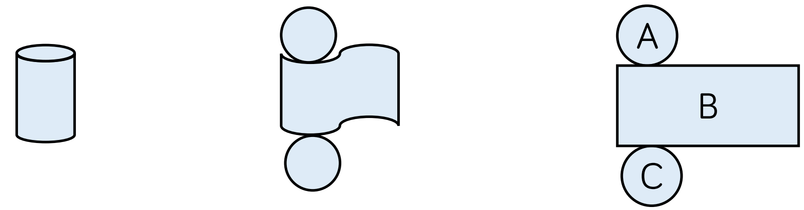
Curved surface	Base	Area
Slant height	Perpendicular height	

Key questions

How many surfaces does a cylinder have? Describe its net.
How do your calculations change if a question concerns an 'open' cylinder?
Which is longer, the slant height or the perpendicular height of a cone? Will this always be the case?

Exemplar Questions

The diagrams below show the formation of the net of a cylinder. Use the diagrams to help find the formula for surface area of a cylinder.

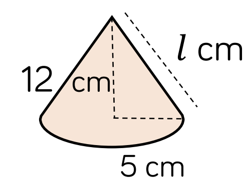


Which cylinder has the greater surface area? Use calculations to justify your answer.

Cylinder A
Height = 9 cm
Radius = 8 cm

Cylinder B
Height = 8 cm
Radius = 9 cm

The curved surface area of a cone is given by the formula $A = \pi r l$, where l is the slant height of the cone.
Work out l and the **total** surface area of the cone shown.



A sweet manufacturer is considering two types of packaging for popping candy:

- A cone of radius 2 cm and height 6 cm.
- A cylinder of radius 2 cm and height 2 cm.

Show that both packages have the same volume and compare their surface areas.

White Rose Maths

Area/Volume of similar shapes

R H

Notes and guidance

Now that students are familiar with the formulae to work with circular shapes, this is a good opportunity for higher tier students to revisit area and volume ratios. Starting with a rectangle and cuboid, students can review length, area and volume scale factor. They can then apply this to cylinders, cones and spheres. Giving opportunities to work 'backwards', by square rooting or cube rooting to find the length of a radius is useful.

Key vocabulary

Scale Factor	Ratio	Proportion
Square	Cube	Root

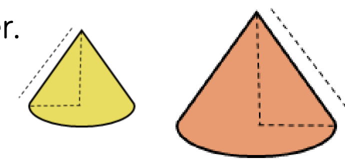
Key questions

How does doubling one length affect the area of a shape? What about doubling all of the lengths?
If one sphere has a radius half the size of another sphere, what's the relationship between their surface areas? What about their volumes?

Exemplar Questions

Two cylinders are similar. The larger cylinder is three times the height of the smaller cylinder.
200 ml of paint are needed to paint the small cylinder.
How much paint is needed to paint the large cylinder?
How many times larger is the volume of the large cylinder compared to the volume of the small cylinder?

The two cones are similar.
The smaller cone has radius 5 cm and the larger cone has radius 20 cm. Is the surface area of the larger cone four times the surface area of the smaller cone? Explain your answer.
The volume of the larger cone is 960 cm^3.
Find the volume of the smaller cone.



Two similar cones have volumes 12π cm^3 and 324π cm^3.
How many times larger is the surface area of the larger cone, compared to the surface area of the smaller cone?

Oil is stored in similar cylindrical drums. The large drum has a base area of π m^2 and the small drum has a base area of 2500π cm^2.
What's the relationship between the radii of the two drums?
How many times more oil can the large drum store than the small drum?