Simultaneous Equations

Year 10

#MathsEveryoneCan

White Rose Maths

	Week 1	Week 2	Week 3	Week 4	Week 5	Week 6	Week 7	Week 8	Week 9	Week 10	Week 11	Week 12
Autumn	**Similarity**						**Developing Algebra**					
	Congruence, similarity and enlargement			Trigonometry			Representing solutions of equations and inequalities			Simultaneous equations		
Spring	**Geometry**						**Proportions and Proportional Change**					
	Angles & bearings		Working with circles		Vectors		Ratios & fractions		Percentages and Interest		Probability	
Summer	**Delving into data**				**Using number**						**Expressions**	
	Collecting, representing and interpreting data				Non-calculator methods		Types of number and sequences		Indices and Roots		Manipulating expressions	

Autumn 2: Developing Algebra

Weeks 7 to 9: Equations and Inequalities	Weeks 10 to 12: Simultaneous Equations
Students will have covered both equations and inequalities at key stage 3, and this unit offers the opportunity to revisit and reinforce standard techniques and deepen their understanding. Looking at the difference between equations and inequalities, students will establish the difference between a solution and a solution set; they will also explore how number lines and graphs can be used to represent the solutions to inequalities. As well as solving equations, emphasis needs to be placed on forming equations from given information. This provides an excellent opportunity to revisit other topics in the curriculum such as angles on a straight line/in shapes/parallel lines, probability, area and perimeter etc. Factorising quadratics to solve equations is covered in the Higher strand here and is revisited in the Core strand in Year 11 National curriculum content covered (Higher content in bold): • consolidate their algebraic capability from key stage 3 and extend their understanding of algebraic simplification and manipulation to include quadratic expressions • translate simple situations or procedures into algebraic expressions or formulae; derive an equation, solve the equation and interpret the solution • select appropriate concepts, methods and techniques to apply to unfamiliar and non-routine problems; interpret their solution in the context of the given problem. • recognise, sketch and interpret graphs of linear functions, • factorising quadratic expressions of the form x^2 + bx + c (Higher only at this stage) • solve quadratic equations algebraically by factorising (Higher only at this stage) • solve linear inequalities in one **{or two}** variable**{s}**, **{and quadratic inequalities in one variable}**; represent the solution set on a number line, **{using set notation and on a graph}**	Students now move on to the solution of simultaneous equations by both algebraic and graphical methods. The method of substitution will be dealt with before elimination, considering the substitution of a known value and then an expression. With elimination, all types of equations will be considered, covering simple addition and subtraction up to complex pairs where both equations need adjustment. Links will be made to graphs and forming the equations will be explored as well as solving them. The Higher strand will include the solution of a pair of simultaneous equations where one is a quadratic, again dealing with factorisation only at this stage. National curriculum content covered (Higher content in bold): • consolidate their algebraic capability from key stage 3 and extend their understanding of algebraic simplification and manipulation to include quadratic expressions • model situations mathematically and express the results using a range of formal mathematical representations, reflecting on how their solutions may have been affected by any modelling assumptions • translate simple situations or procedures into algebraic expressions or formulae; derive an equation (or two simultaneous equations), solve the equation(s) and interpret the solution • select appropriate concepts, methods and techniques to apply to unfamiliar and non-routine problems; interpret their solution in the context of the given problem. • solve two simultaneous equations in two variables (linear/linear **{or linear/quadratic}**) algebraically; • recognise, sketch and interpret graphs of linear functions and quadratic functions.

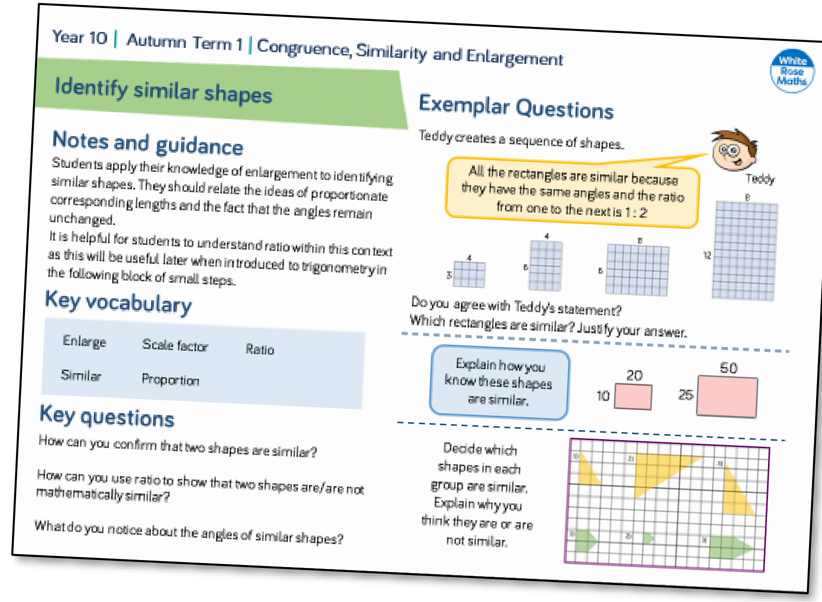
Why Small Steps?

We know that breaking the curriculum down into small manageable steps should help students to understand concepts better. Too often, we have noticed that teachers will try and cover too many concepts at once and this can lead to cognitive overload. We believe it is better to follow a "small steps" approach.

As a result, for each block of content in the scheme of learning we will provide a "small step" breakdown. ***It is not the intention that each small step should last a lesson – some will be a short step within a lesson, some will take longer than a lesson.*** We would encourage teachers to spend the appropriate amount of time on each step for their group, and to teach some of the steps alongside each other if necessary.

What We Provide

- Some ***brief guidance*** notes to help identify key teaching and learning points
- A list of ***key vocabulary*** that we would expect teachers to draw to students' attention when teaching the small step,
- A series of ***key questions*** to incorporate in lessons to aid mathematical thinking.
- A set of questions to help ***exemplify*** the small step concept that needs to be focussed on.


Year 10 | Autumn Term 1 | Congruence, Similarity and Enlargement

Identify similar shapes

Notes and guidance

Students apply their knowledge of enlargement to identifying similar shapes. They should relate the ideas of proportionate corresponding lengths and the fact that the angles remain unchanged.

It is helpful for students to understand ratio within this context as this will be useful later when introduced to trigonometry in the following block of small steps.

Key vocabulary

Enlarge, Scale factor, Ratio, Similar, Proportion

Key questions

How can you confirm that two shapes are similar?

How can you use ratio to show that two shapes are/are not mathematically similar?

What do you notice about the angles of similar shapes?

Exemplar Questions

Teddy creates a sequence of shapes.

All the rectangles are similar because they have the same angles and the ratio from one to the next is 1 : 2

Do you agree with Teddy's statement? Which rectangles are similar? Justify your answer.

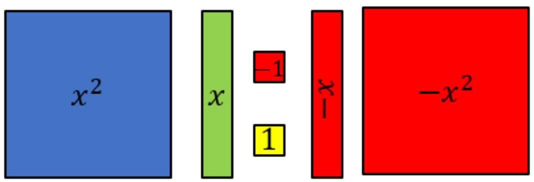
Explain how you know these shapes are similar.

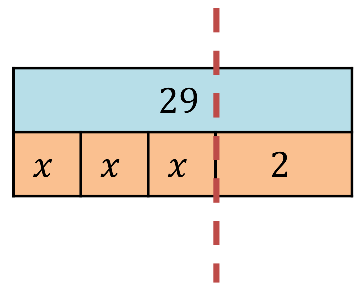
Decide which shapes in each group are similar. Explain why you think they are or are not similar.

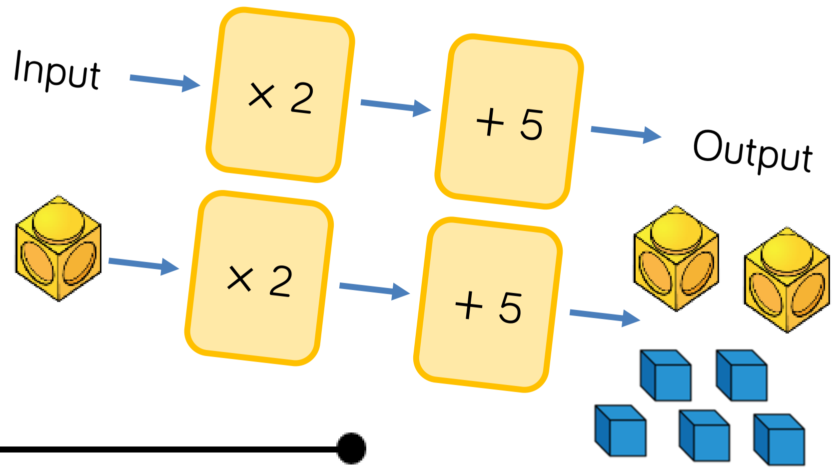
- These include reasoning and problem-solving questions that are fully integrated into the scheme of learning. Depending on the attainment of your students, you many wish to use some or all of these exemplars, which are in approximate order of difficulty. Particularly challenging questions are indicated with the symbol .
- For each block, we also provide ideas for key representations that will be useful for all students.

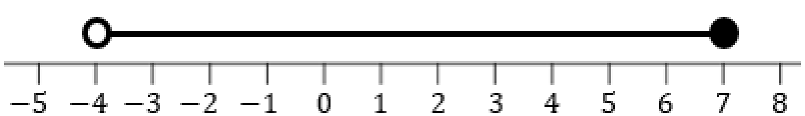
Some of the small steps are in **bold** and labelled with H to indicate this is Higher tier GCSE content. We would encourage teachers to use these with as many students as possible – if you feel your class can access any particular small step, then please include it in your planning. Steps that review content covered at Key Stage 3 are labelled R .

Key Representations









Here are a few ideas for how you might represent algebraic expressions and the solutions of simultaneous equations.

Cups, cubes and elastic bands lend themselves well to representing an unknown, whereas ones (from Base 10) and counters work well to represent a known number. Be careful to ensure that when representing an unknown, students use equipment that does not have an assigned value – such as Base 10 equipment and dice.

Bar models are useful to support the forming of equations and also help students to make sense of the approach to a solution. Algebra tiles are also very powerful for this and help to make sense of factorising quadratics, alternate representations are very effective in ensuring all students, including higher attaining, make sense of the mathematical structures.

Simultaneous Equations

Small Steps

- Understand that equations can have more than one solution
- Determine whether a given (x, y) is a solution to a pair of linear simultaneous equations
- Solve a pair of linear simultaneous equations by substituting a known variable
- Solve a pair of linear simultaneous equations by substituting an expression (1) & (2)
- Solve a pair of linear simultaneous equations using graphs
- Solve a pair of linear simultaneous equations by subtracting equations
- Solve a pair of linear simultaneous equations by adding equations
- Use a given equation to derive related facts R

H Denotes Higher Tier GCSE content

R Denotes 'review step' – content should have been covered at KS3

Simultaneous Equations

Small Steps

- Solve a pair of linear simultaneous equations by adjusting one equation
- Solve a pair of linear simultaneous equations by adjusting both equations
- Form a pair of linear simultaneous equations from given information
- Form and solve pair of linear simultaneous equations from given information
- **Determine whether a given (x, y) is a solution to both a linear and quadratic equation** H
- **Solve a pair of simultaneous equations (one linear, one quadratic) using graphs** H
- **Solve a pair of simultaneous equations (one linear, one quadratic) algebraically** H
- **Solve a pair of simultaneous equations involving a third unknown** H

H Denotes Higher Tier GCSE content

R Denotes 'review step' – content should have been covered at KS3

More than one solution

Notes and guidance

Students explore equations that have more than one possible solution. Students should use different types of numbers when finding these solutions (e.g. negatives, decimals, fractions). Building on this, students think about what else is needed to reduce to just one solution. This leads into the idea of requiring two equations and hence into the concept of simultaneous equations.

Key vocabulary

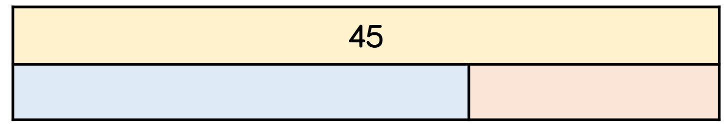
Possible	Solutions	Infinite
Finite	Variables	Equation

Key questions

What possible solutions are there? Are there an infinite number of solutions? Why/Why not?
What else do we need to determine just one solution?
Can you think of a second equation that would help?
Why do you need 2 equations to find each variable? How many equations would be needed if you had 3 variables?

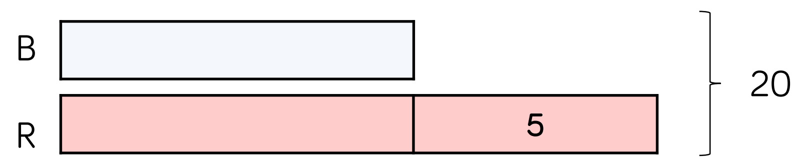
Exemplar Questions

Two numbers add together to give 45. One number is bigger than the other. List some possible solutions. Compare these as a class. Is there one pair of values that must be true? Explain your answer.



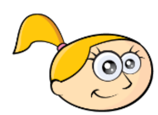
There are 20 red and blue counters in a bag. There are  more red counters than blue counters.

Explain why this bar model must be incorrect.



How many more red counters than blue counters could there be?

If $x + y = 10$, list all possible positive integer values for x and y.



If we also know that the x is 4 more than y, then we can only find one pair of values for x and y.

Eva is correct. Why?
What must the values of x and y be?

Is (x, y) a solution?

Notes and guidance

Students may need practice substituting (including with negative numbers) before attempting this small step. Use of formulae for area and perimeter can be interleaved here.
Students then substitute values into equations to work out whether or not they have a possible solution. They understand that there is one possible solution when two equations are given in terms of two variables.

Key vocabulary

Solution	Substitute	Equation
Variable	Verify	

Key questions

Are there any other possible dimensions for a (e.g.) rectangle, parallelogram, trapezium with area……?
What if I now tell you that the perimeter is…..?

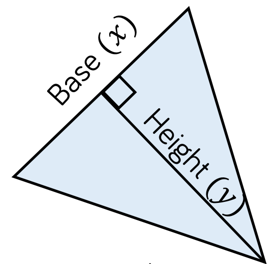
Why can two variables be the same value?

Why is there only one solution to two equations containing two variables?

Exemplar Questions

A straight line has the equation $y = 3x + 5$
Show that the point (1, 8) lies on this line.
Show that the point (2, 7) does not lie on this line.
Show that $x = 3$ and $y = 14$ is one solution to the equation.
Does the point (3, 14) lie on this line?

The area of the triangle is 50 cm^2
Complete the table exploring possible values for x and y.



Not to scale

Base (x)	Height (y)	True/False
10	5	
10	10	
25	2	
2.5	40	
–10	–10	

$$j^2 + k = 6$$

Mo thinks that $j = 2$ and $k = 2$ is a solution for this equation.
Annie says that can't be right as j and k can't be the same value.
Who do you agree with and why?
Show that $j = -3, \ k = -3$ is also a solution to this equation.

Is $x = 5$ and $y = 3$ a solution to both equations?
Show workings to justify your answer.
Does the point (5, 3) lie on both of the lines?

$$y = 13 - 2x$$
$$3x - 2y = 9$$

Substituting a known variable

Notes and guidance

Before starting, students need to review solving equations. Modelling substitution and solving equations is key. There is opportunity to interleave aspects of measure (e.g. $P = 2l + 2w$). Using bar models to begin with will support algebraic thinking. The students go onto realise that there may be more than one way of finding a solution if presented with two related equations.

Key vocabulary

Substitute	Solve	Solution
Variable	Unknown	Inverse

Key questions

Which variable can we substitute for?

Can you write an equation with one unknown variable?

What are the steps in solving this equation?

If I have two related equations, does it matter which one I substitute into and then solve?

Exemplar Questions

Three hops (h) and two jumps (j) have a total length of 39 m.

39				
h	h	h	j	j

A hop is 6 m in length. How long is a jump?
How long would two hops and three jumps be?

A straight line has equation $2x + 10 = y$

Amy wants to find possible solutions to this equation.
She knows that $x = 5$ Use this information to find the value of y.
Does the point with coordinates (5, 20) lie on this straight line?
At another point on the line, $y = 30$ Use this information to find x.

Teddy and Whitney are working out the value of h. Whose method do you like best? Why might it be useful to do both?

$h + j = 25$
$3h - j = 27$
$j = 12$

Teddy

If we know $j = 12$, then we can use the first equation to work out h.
$h + 12 = 25$
$h = 13$

Whitney

If we know $j = 12$, then we can use the second equation to work out h.
$3h - 12 = 27$
$3h = 39$
$h = 13$

Substituting into an expression (1)

Notes and guidance

This small step introduces the idea of substituting one equation into a second equation and is split into two parts. Double-sided counters could be used so that students can physically make the substitution. Students might then use pictorial representations before attempting the abstract substitution. At this stage, students are not rearranging in order to make the substitution.

Key vocabulary

Substitute	Substitution	Solve

Key questions

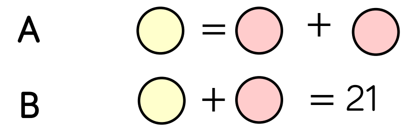
What object/letter can I substitute for?

What can I replace it with?

Why does this help me to find the value of one of the variables?

How can you check your answers?

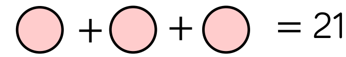
Exemplar Questions



Annie looks at equation **A** and says: “Every yellow counter is worth two red counters”

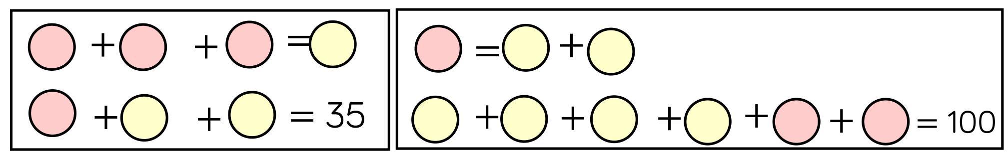
Annie

Annie looks at **B** and says: “I can swap one yellow counter for two red counters”



Annie says “If 3 red counters are worth 21, then each red counter is worth 7”. Using **B**, work out the value of the yellow counter.

Using the same method, find the value of the red and the yellow counter in each of these cases.



Equation 1	□	=	△ + 3
Equation 2	□ □ △ △ △	=	26
In equation 2 I can substitute △ + 3 for each □			
	△ + 3 △ + 3 △ △ △	=	26

Rosie

How did Rosie know what to substitute?

Simplify the new equation, to work out the value of △

Use another equation to work out the value of □

Which equation did you use to find □ ? Why?

Substituting into an expression (2)

Notes and guidance

In this second part of the step, higher tier students might now explore rearranging an equation to make a substitution. Finding the subject of the formula may need revising. Teachers might then emphasise the importance of checking directed number when substituting (particularly when substituting an expression for x into $-x$), and the choice of which letter to substitute for.

Key vocabulary

Subject of the formula	Rearrange
Simultaneous equations	Substitute

Key questions

What happens if we substitute (e.g.) $x = 7 - y$ into $y - x = 5$?
Which equation is easiest to rearrange?
Which variable is easiest to make the subject of the formula?
Which equation do we now need to substitute into?

Exemplar Questions

Rearrange each of the equations to make x the subject.

$x - 3y = 6$ $\quad$ $2x - 3y = 6$ $\quad$ $3y - 2x = 6$

$$x + y = 7$$
$$y - x = 5$$

Rearranging the 1st equation:
$x = 7 - y$
Substitute into $y - x = 5$

Rearranging the 2nd equation:
$y = 5 + x$
Substitute into $x + y = 7$

Which method is correct? Explain your answer.
Try both methods. Which was easiest to do?
Choose a method and complete it to find the value of x and y.

Jack 

$$k = -9 - 2m$$
$$k = 3m + 11$$

Jack starts solving this by rearranging the first equation.

$$k + 2m = -9$$
$$2m = -9 - k$$
$$m = \frac{-9 - k}{2}$$

Is there an easier first step that Jack could have taken?
Solve the simultaneous equation using a more efficient method.
How can you check your answers?

Solve by using graphs

Notes and guidance

Students learn that the intersection point of two straight lines represents the solution to a pair of linear equations, comparing graphical and algebraic methods. It's important that teachers emphasise that it is the value of x and the value of y that give the solution, rather than the coordinate. Teachers could extend this by exploring why some pairs of linear equations do not have any solutions (parallel lines).

Key vocabulary

Intersect	Coordinate	Solution
Substitution	Meet	

Key questions

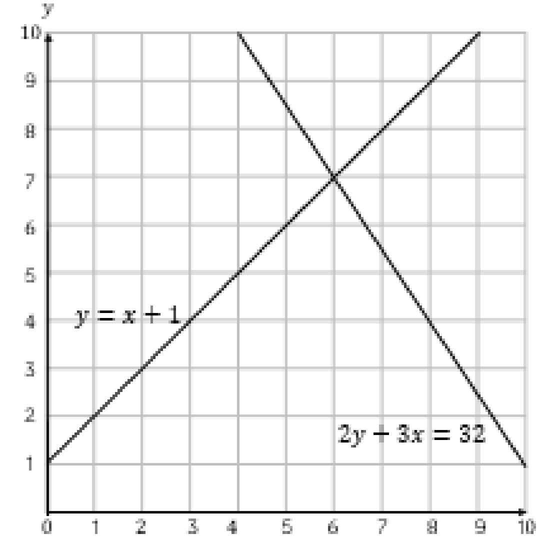
What does intersect mean? Where go graphs 'meet'?
What is true about the coordinates of the point where two lines meet? How do they relate to the equations?
Explain why the intersection point represents the solution to a pair of equations.
Can there be more than one pair of solutions to this pair of simultaneous equations?

Exemplar Questions

Tommy draws the graphs of

$$y = x + 1$$
$$2y + 3x = 32$$

a) Write down the coordinate of the point where the lines meet.
b) What is the value of x? What is the value of y?
c) Solve the simultaneous equation using substitution.
d) What do you notice?



Complete the tables.

$y = 2x$

x	0	3	5
y			

$x + y = 9$

x	0	3	5
y			

Use your tables to draw the graph of $y = 2x$ and $x + y = 9$

Use your graph to solve:

$$y = 2x$$
$$x + y = 9$$

There might be more than one possible solution

Ron

Explain why Ron is incorrect for this pair of equations.
Could there be a pair of equations with more than one solution, or no solutions? Explain you answer.

Solve by subtracting equation (1)

Notes and guidance

This step is split into two; firstly looking at subtracting positive number. Teachers might introduce this using concrete resources allowing students to physically subtract two equations. Bar models also clearly show the difference between two equations. Once students understand why subtracting eliminates a variable they can attempt abstract simultaneous equations, include answers which are zero, negative or non-integer.

Key vocabulary

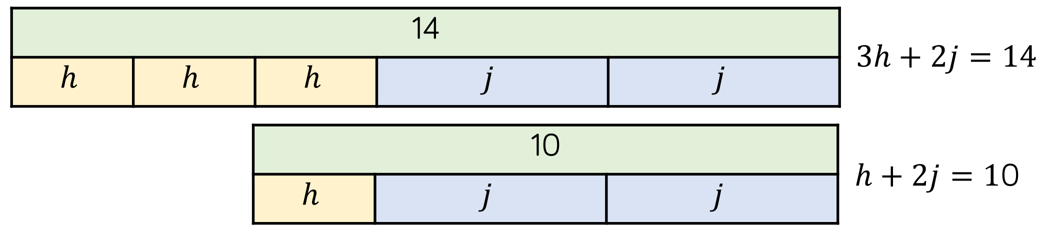
Subtract	Negative	Eliminate
Variable		

Key questions

Why is it useful to 'eliminate' one of the variables?
Which equation do we substitute into? Does it matter? Why/why not?
Does it matter if we subtract equation 1 from equation 2 or equation 2 from equation 1? Which is easier to do?
How can we check our answers?

Exemplar Questions

Amir is comparing the lengths of hops (h) and jumps (j)



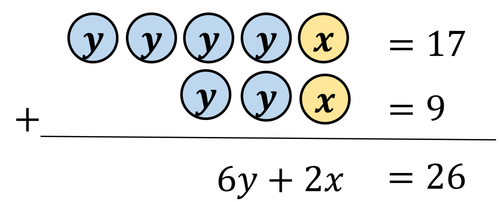
From the bar models, Amir works out,

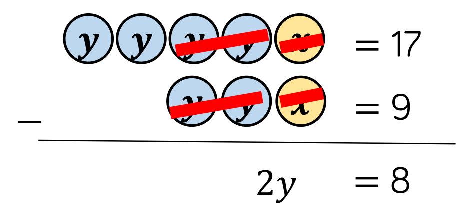
$$2h = 4$$

How did he work this out? Now work out the values of h and j

$4y + x = 17$ A
$2y + x = 9$ B

How do the diagrams below link to equations A and B?





Which would be more beneficial to help us solve these equations, to add the two equations or to subtract the two equations? Why? Now find the value of y and use this to find the value of x.

Solve the following simultaneous equations by subtracting the equations.

$2x + y = 17$
$x + y = 10$

$3x + y = -2$
$3x + 2y = 2$

$4x - 2y = 11$
$4x + y = 18.5$

Solve by subtracting equation (2)

Notes and guidance

Careful revision of subtracting a negative will be required at the start of this continuation of the step. Revising solving single equations that involve negatives will also be useful. Students then consider why subtraction of the two equations eliminates a variable, before considering the pitfalls of subtracting equations containing negatives. Students will be able to generalise (when do we subtract the two equations – what do you notice?)

Key vocabulary

Expression	Equation	Eliminate
Subtract	Negative	Solve

Key questions

What happens when we subtract a negative number?

What do you notice about the type of coefficients in these simultaneous equations, where the first step is to subtract?

How can we check our answers for both original equations?

Exemplar Questions

Simplify the following expressions.

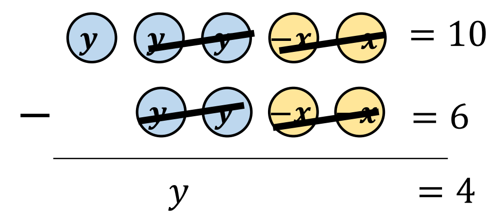
$-3x - (-3x)$

$-5y + 3x - (-5y) + 6x$

$4y - x - 2y - (-x)$

The diagram represents the two equations in the purple box.

$3y - 2x = 10$
$2y - 2x = 6$



Explain why $-2x - (-2x) = 0$

Is it useful to add the two equations together. Why or why not?

Now solve the simultaneous equations and check your answers.

Annie, Tommy and Dexter are attempting to solve this pair of simultaneous equations. They all start by subtracting the two equations. **Two** of them have made a mistake.

Who has made mistakes? What mistake did they make?

$-6x - 7y = 9$

Annie

$3y = 9$

Tommy

$7y = 63$

Dexter

One person has started correctly.
Continue their solution to find the values of x and y.

Solve by adding equations

Notes and guidance

By considering the simplification of expressions, students understand how to make zero using addition. They build on this to solve simultaneous equations involving negative or non-integer solutions. They progress to consider which equation is more efficient when substituting to find the second solution. It's also important to consider equations where it might be easier to rearrange before adding.

Key vocabulary

Expression	Equation	Eliminate
Subtract	Negative	Solve

Key questions

What happens when we add with negative numbers?

Which equation should we now substitute in to? Why?

Does it matter which equation we substitute into?

Would it help to rearrange the equations first?

Exemplar Questions

Simplify the following expressions.

$-2x + 2x$

$5y + 3x + (-5y) + 6x$

$4y - x - 2y + (-x)$

$3x + 2y = 16$ A

$6x - 2y = 2$ B

Which variable, x or y, would you try to eliminate? Why?

Ron decides to eliminate y.

Should he add equations A and B or subtract them? Why?

Show that $x = 2$

Ron substitutes this into equation A.

$6 + 2y = 16$

Show that $y = 5$

Ron wants to check his answer. He substitutes his values for x and y back into equation A. Why isn't this sensible?

Check Ron's values by substituting into equation B.

For each of the pairs of simultaneous equations., decide whether you would add or subtract the equations and then solve each pair of simultaneous equations.

$3x + 2y = 24$
$3x - 5y = -18$

$6x + 2y = 12$
$6x - 2y = 0$

$3w + 2v = 2$
$w = -6 + 2v$

Related facts from an equation R

Notes and guidance

Teachers might start this small step by reviewing where students have met the concept of equivalence before (for example, ratios, fractions). It's important to ensure that students understand that equivalent equations have the same solutions. This step relates closely to deriving related number facts e.g. working out 4×17 from doubling 2×17, and this makes a good introduction.

Key vocabulary

Equivalent	Solution	Coefficient
Variable	Multiplier	

Key questions

What happens when we substitute our original solutions into the equivalent equations? Why does this happen?

How can I generate an equivalent equation?

What happens if I divide the equations by a number instead of multiplying? Do the solutions change now?

Exemplar Questions

Complete the diagram by replacing the questions marks.

$3x + y = 5$

$\times 4$ → $12x + 4y = 20$

$\times 3$ → $?x + ?y = ?$

$\times 2$ → $?x + 2y = 10$

$\times 5$ → $15x + ?y = ?$

Is $x = 1$ and $y = 2$ a solution for each equation? Why?

Now write down four more equations that are equivalent to $3x + y = 5$

Amir and Dora are discussing the following pair of equations.

$4x + 3y = 23$ $\qquad$ $8x + 6y = 46$

Amir: The equations look different so x and y must be different in each equation.

Dora: The equations are equivalent. This means that the value of x is the same in both equations.

Who is correct? Why? If $x = 5$, work out y. Are these values of x and y solutions to both equations? Explain your answer.

Alex uses the equation $6s - 2t = 4$ to form three other equations. She's made some mistakes. Find her mistakes and correct them.

$60s - 20t = 4$ $\qquad$ $18s - 6t = 8$ $\qquad$ $12s - 2t = 8$

Solve by adjusting one equation

Notes and guidance

Bar models are a good way of demonstrating why equal coefficients of one of the variables is necessary when we are solving by elimination. It is useful to provide the abstract equation alongside each bar model to support conceptual understanding of the method. Teachers should also discuss whether to make the coefficients of x the same, y the same, and whether it matters.

Key vocabulary

Coefficient	Variable
Multiplier	Solve

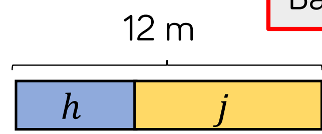
Key questions

Why do we need the coefficient of one of the variables to be the same in both equations? How does this help us to solve the equations?
Once we know one variable, how do we find the other?
Which equation do we multiply and why? Is there more than one way of making the coefficients the same?

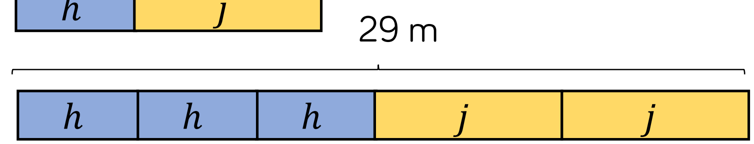
Exemplar Questions

Whitney draws a bar model to represent the following problem.
A hop and a jump have total length 12 m
Three hops and two jumps have total length 29 m

Bar Models | Equation

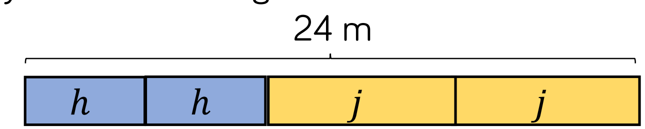


$h + j = 12$ (A)



$3h + 2j = 29$ (B)

Whitney doubles the length of her first bar.



$2h + 2j = 24$ (C)

How do these bar models show that $h = 5$?
Show that (B) − (C) also gives $h = 5$
Now find the length of a jump.

Another way to find the length of a hop is to multiply equation by 3
Then you have the same number of hops.

Draw a bar model to represent this. Compare your bar model with B.
Find the length of a hop and a jump using this method.

In each of the following, multiply one equation to make the coefficient of x or y the same.

$3x + 2y = 4$
$4x + y = 9$

$3x + 2y = 14$
$x + y = 5$

$x + y = 6$
$3x - 3y = 0$

White Rose Maths

Solve by adjusting both equations

Notes and guidance

To begin with, teachers may need to guide students in choosing appropriate multipliers; a review of LCM will assist this. Students will realise that it doesn't matter which variable they focus on when making coefficients the same, but should consider which variable is easier (for example, avoiding negatives). Choosing whether to add or subtract should again be reinforced.

Key vocabulary

Lowest Common Multiple	Coefficient
Variable	Multiplier

Key questions

When making the coefficients the same, which variable should we choose?

Why would we choose to multiply by smaller numbers wherever possible?

How do we know whether to add or subtract? If we have a choice, which is easier?

Exemplar Questions

Find the lowest common multiple of the following.

8 and 12 | 6 and 10 | 4 and 5

Amir and Rosie are solving this pair of simultaneous equations.

$2x + 3y = 39$ (A)

$5x - 2y = -7$ (B)

Multiply equation **A** by 2
Multiply equation **B** by 3
Now add the two new equations.

Multiply equation **A** by 5
Multiply equation **B** by 2
Now subtract the two new equations.

Explain why both Amir's and Rosie's methods work.
Find the solutions using both methods. Which did you find easier?

Tommy multiplies equation **A** by 8 and equation **B** by 12
Write down two smaller multipliers he could have used.
To eliminate x, should Tommy add the two equations or subtract them?
Explain your answer.
Solve the simultaneous equations.

$12x - 5y = -22$ (A)

$-8x + 4y = 16$ (B)

Dexter wants to eliminate t. What should he multiply by? He now subtracts the two equations. Will this eliminate t?
By eliminating t, solve the simultaneous equation.
Check your answer by substituting your values back into one equation.

$-6s + 10t = 11$ (A)

$10s - 6t = 3$ (B)

Form a pair of linear equations

Notes and guidance

Students might start by using counters to solve wordy problems. This will help them to formulate the algebraic equations. Students often get confused about forming equations involving 'more than' or 'doubling', placing the addition/multiplication on the wrong side of the equation. This will need exploring by testing values. Students must give final answers in the context of the question.

Key vocabulary

Formulate	Variable	Context
Equation		

Key questions

Tell me one thing that you know. How could we write this as an equation? What could we use to represent the variable?

How could we check whether the equation we have written down is correct?

Does your answer relate to the context of the question?

Exemplar Questions

In the car park, there are 18 vehicles. Some are cars and some are bikes. There are 60 wheels in total in the car park.
Dora decides to use algebra to solve the problem.

Let c = number of cars and b = number of bikes

Dora

Complete the equations.

$b + c = \square$

$2b + \square c = 60$

Alex and Jack have £10.00 between them. Alex has £1.80 more than Jack. Let a = amount of money Alex has and let j = amount of money Jack has.
Write an equation to show how much money they have in total.
Using the given information, Alex and Jack have written another equation:

Alex

$a = j + 1.8$

Jack

$a + 1.8 = j$

Who's right and why?

Miss Rose is twice the age of her little brother.
If x = age of Miss Rose and y = age of her brother, which of these equations is correct?

$2x = y$

$x = 2y$

Check your answer by substituting in a value for Miss Rose's age.
The total of their ages is 48. Write down a second equation.

Form & solve two linear equations

Notes and guidance

Building from the last step, students continue to develop their skills in forming equations in conjunction with practising solving them. Students may need to be provided with scaffolding when first attempting to form and then solve two linear equations. This should be gradually removed. Interleaving other topic areas, such as shape, works well in this small step.

Key vocabulary

Formulate	Variable	Context
Equation	Solve	Solution

Key questions

Tell me one thing that you know. How could we write this as an equation?
When making the coefficients the same, which variable should we choose? How do we know whether to add or subtract the equations?
Does your answer relate to the context of the question?

Exemplar Questions

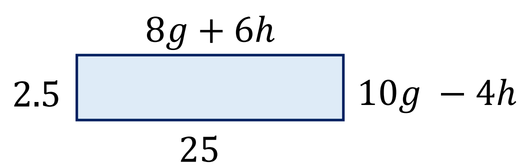
1000 tickets are sold for a concert.
Adult tickets are £10 and Child tickets are £6
£7304 was collected through ticket sales.

Ron writes down these two equations.
What do you think x and y represent?
What mistakes has Ron made?
Correct the equations.

$$10x + 6y = 1000$$
$$x + y = 7304$$

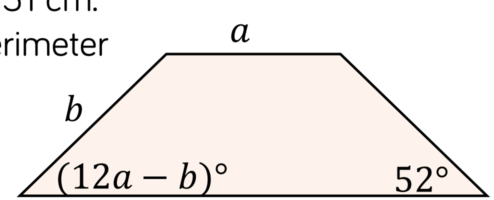
How many adult tickets and how many child tickets were sold?

What do you know about the side lengths of a rectangle?
Use this to form two equations involving g and h.



Solve this simultaneous equation to find g and h.

The base of an isosceles trapezium is double the length of side a.
The perimeter of the trapezium is 31 cm.
Write down an equation for the perimeter of the trapezium.
Look at the marked angles.
Write down another equation relating a and b.
Find the length of the base of the trapezium.



Is (x, y) a solution? H

Notes and guidance

Teachers might start with a quick review of substituting negative numbers into quadratic expressions. Students also need to be aware that when substituting into expressions such as $2x^2$, they calculate x^2 before multiplying by 2

Recognising linear and quadratic equations may also need reviewing. It's then important to make the link between coordinates that are on both curve and line & the solution to the simultaneous equations.

Key vocabulary

Quadratic	Curve	Linear	Coordinate
Solution	Substitute	Square	

Key questions

What's the same and what's different about the equations of a straight line and the equations of a curve?

How can we recognise which equation will produce a curve?

Exemplar Questions

Whitney is thinking about this equation of a curve. $y = x^2$

She uses substitution to find out whether (−1, 1) is on the curve.

If $x = -1$ then $y = -1^2$ and so $y = -1$

She concludes that $(-1, 1)$ isn't on the curve.
Whitney is wrong. Why?

- Is $(1, 1)$ on the curve?

Whitney looks at this pair of equations:

$$y = x^2$$
$$y = x$$

- Is $(1, 1)$ on both the curve and the line? What about $(-1, 1)$?

Whitney thinks that (0, 0) is a solution to both equations. Show that she is correct.

State which of the following coordinates are,

- On the curve only
- On the line only
- On both the curve and the line
- On neither the curve nor the line

$$y = 2x^2$$
$$y = 6x - 4$$

Show your workings to justify each answer.

$(1, 2)$	$(-1, -2)$	$(2, 16)$	$(2, 8)$
$(1, 4)$	$(-1, 2)$	$(-1, -10)$	$(-1, 10)$

Write down the values of x and y that are solutions to both equations.

Solving graphically

H

Notes and guidance

This is a good opportunity to revise how we know whether an equation represents a curve or a straight line. Students may need reminding about how to draw a smooth curve. Students then make the link that the solution is represented by the intersection point. It's important that teachers emphasise the difference between how the solution is presented, $x = \_\_, y = \_\_$ and the intersection point (x, y)

Key vocabulary

Intersection	Solution	Linear
Non-linear	Quadratic	Curve

Key questions

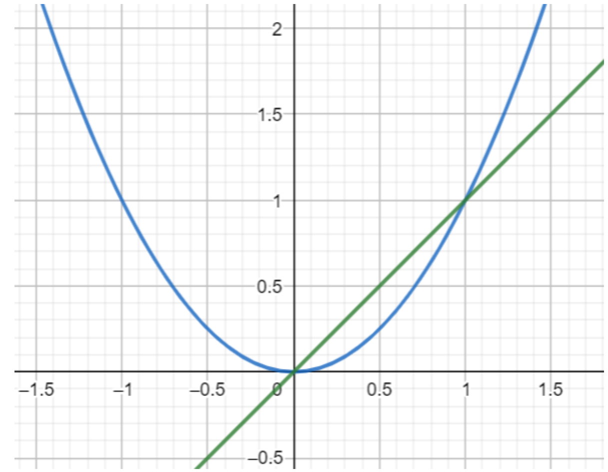
How can we recognise a quadratic equation?
What does this look like on a graph?
Why don't we use a ruler to sketch the curve?
How do we represent the point of intersection ?
How is this presentation different to the presentation of the solution of the pair of equations?

Exemplar Questions

Tommy draws the graph of:

$$y = x^2$$
$$y = x$$

Label the curve and the line with the correct equations.



- Write down the coordinates of the point where the curve and the line intersect. What are the values of x and y?
- Are these pairs of values solutions to both equations? Substitute the values into the equations to check.
- How do you know that there will only be two solutions?

Complete both table of values.

$y = x^2 + 2$

x	−3	−2	−1	0	1	2	3	4	5
y	11					6			

$y = 2x + 10$

x	−3	0	3
y			

Use your tables to draw the graphs of $y = x^2 + 2$ and $y = 2x + 10$

Use your graphs to solve,

$$y = x^2 + 2$$
$$y = 2x + 10$$

Write your solutions in the form $x = \_\_, y = \_\_$

Solving algebraically (1)

H

Notes and guidance

This step is split into two parts. It is useful to start by revising factorising and solving quadratics. Students consider simultaneous equations where both are in the form $y =$
This allows them to gain confidence in manipulating and solving quadratics, before going onto more complex equations. Finally, to find the corresponding value of y, teachers should emphasise that it is easier to substitute back into the linear equation.

Key vocabulary

Factorise	Rearrange	Solve
Linear	Quadratic	

Key questions

Why does it make sense to substitute for y in these cases?
Could I substitute for x instead? Why is this less efficient for these equations?
Why is it easier to substitute back into the linear equation to find the value of y?
How can we check the answers?

Exemplar Questions

Factorise
$x^2 - 2x - 3$ $\quad x^2 - 6x + 8$ $\quad x^2 + 5x - 24$

Solve
$x^2 - 2x - 3 = 0$ $\quad x^2 - 6x + 8 = 0$ $\quad x^2 + 5x - 24 = 0$

Mo is working on this pair of simultaneous equations.

$y = x^2$ (A)
$y = 2x + 3$ (B)

I can substitute y for x^2 in equation **B**:
$x^2 = 2x + 3$
I can rearrange this so that I can solve it:
$x^2 - 2x - 3 = 0$

Complete Mo's solution by finding the values of x.

Find the corresponding values of y by substituting your values of x into the linear equation (**B**).

Write down two pairs of solutions.

By equating the two expressions for y, show that the pair of solutions to this simultaneous equation are:,
$x = 4, y = 20$ and $x = 2, y = 8$

$y = x^2 + 4$
$y = 6x - 4$

Solving algebraically (2)

H

Notes and guidance

Teachers may wish to start this continuation of a small step by expanding brackets e.g. $(y-3)^2$

Students are now encouraged to think whether it is easiest to rearrange the linear equation first, or whether they can make a direct substitution. They understand that they can substitute for either x or y and decide which is the most efficient.

Key vocabulary

Rearrange	Linear	Substitute
Quadratic	Solve	Solution

Key questions

Could I substitute for x? How could I do this?

Could I substitute for y? Do I need to rearrange the linear equation first. How could I do this?

Which method is most efficient?

Exemplar Questions

Dora is solving the simultaneous equations.

(A) $x = y - 3$
(B) $x^2 + 2y = 9$

Here is Dora's first step: $(y-3)^2 + 2y = 9$

Explain what Dora has done in this first step.

$$y^2 - 4y = 0$$

Now solve this equation to show that $y = 0$, $y = 4$

Complete the solution by substituting these values into equation A.

Tommy is solving the simultaneous equations.

(A) $y = 2x - 5$
(B) $x^2 + y^2 = 10$

He substitutes $y = 2x - 5$ into equation B: $x^2 + (2x-5)^2 = 10$

Tommy simplifies this: $x^2 + 4x^2 - 25 = 10$

Where has Tommy gone wrong?

Correct Tommy's work and go on to solve the simultaneous equation.

Is (3, 1) an intersection point of the two graphs? How do you know?

Alex is solving this simultaneous equation.

(A) $y = x^2$
(B) $5x = 24 - y$

She starts by rearranging equation B:

$$y = 24 - 5x$$

She substitutes this into equation A: $24 - 5x = x^2$

Dexter thinks it's easier to substitute $y = x^2$ into equation B:

$$5x = 24 - x^2$$

Which approach do you prefer? Why?

Solve the simultaneous equations.

Solve with a third unknown H

Notes and guidance

A good starting point would be to ask what's the same and what's different when comparing the usual simultaneous equations with two variables to these questions involving a constant. They may need guidance in understanding how to write x and y 'in terms of' a constant. They can check by substituting a values for the constant and verifying their solutions work in the original equations.

Key vocabulary

Variable	Constant	Simplest Form
In terms of		

Key questions

What is a constant?

If I replaced the constant with a number would you be able to solve the pair of equations?

What does 'give your answers in terms of…' mean?
What is meant by 'simplest form'?

Exemplar Questions

In the simultaneous equations, a is a constant.

$x + 4y = 22a$ (A)
$y = 2x + a$ (B)

Complete the solution.

Step 1: Substitute equation **B** into equation **A**:

$$x + 4(\square) = 22a$$

Step 2: Find x in terms of a:

$$x + 8x + 4a = 22a$$
$$\square x + 4a = 22a$$
$$9x = \square a$$
$$x = \square a$$

Step 3: Substitute $x = 2a$ into equation **A**:

$$\square + 4y = 22a$$

Step 4: Solve this to find y:

$$4y = \square\ a$$
$$y = \square\ a$$

In the simultaneous equation, p is a constant.
Solve the simultaneous equations, giving your answers in terms of p in their simplest form.

$4x + y = 5p$ (A)
$y = 2x + 2p$ (B)

When Dexter attempted this question, he left his answer as $2x = p$

Why is this incorrect?

Check your solutions in the equations if $p = 6$