Equations and Inequalities

Year 10

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White Rose Maths

	Week 1	Week 2	Week 3	Week 4	Week 5	Week 6	Week 7	Week 8	Week 9	Week 10	Week 11	Week 12
Autumn	**Similarity**						**Developing Algebra**					
	Congruence, similarity and enlargement			Trigonometry			Representing solutions of equations and inequalities			Simultaneous equations		
Spring	**Geometry**						**Proportions and Proportional Change**					
	Angles & bearings		Working with circles		Vectors		Ratios & fractions		Percentages and Interest		Probability	
Summer	**Delving into data**				**Using number**						**Expressions**	
	Collecting, representing and interpreting data				Non-calculator methods		Types of number and sequences		Indices and Roots		Manipulating expressions	

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Autumn 2: Developing Algebra

Weeks 7 to 9: Equations and Inequalities

Students will have covered both equations and inequalities at key stage 3,and this unit offers the opportunity to revisit and reinforce standard techniques and deepen their understanding. Looking at the difference between equations and inequalities, students will establish the difference between a solution and a solution set; they will also explore how number lines and graphs can be used to represent the solutions to inequalities.
As well as solving equations, emphasis needs to be placed on forming equations from given information. This provides an excellent opportunity to revisit other topics in the curriculum such as angles on a straight line/in shapes/parallel lines, probability, area and perimeter etc.
Factorising quadratics to solve equations is covered in the Higher strand here and is revisited in the Core strand in Year 11

National curriculum content covered (Higher content in bold):

- consolidate their algebraic capability from key stage 3 and extend their understanding of algebraic simplification and manipulation to include quadratic expressions
- translate simple situations or procedures into algebraic expressions or formulae; derive an equation, solve the equation and interpret the solution
- select appropriate concepts, methods and techniques to apply to unfamiliar and non-routine problems; interpret their solution in the context of the given problem.
- recognise, sketch and interpret graphs of linear functions,
- factorising quadratic expressions of the form x^2 + bx + c (Higher only at this stage)
- solve quadratic equations algebraically by factorising (Higher only at this stage)
- solve linear inequalities in one **{or two}** variable**{s}**, **{and quadratic inequalities in one variable};** represent the solution set on a number line, **{using set notation and on a graph}**

Weeks 10 to 12: Simultaneous Equations

Students now move on to the solution of simultaneous equations by both algebraic and graphical methods. The method of substitution will be dealt with before elimination, considering the substitution of a known value and then an expression. With elimination, all types of equations will be considered, covering simple addition and subtraction up to complex pairs where both equations need adjustment. Links will be made to graphs and forming the equations will be explored as well as solving them.
The Higher strand will include the solution of a pair of simultaneous equations where one is a quadratic, again dealing with factorisation only at this stage.

National curriculum content covered (Higher content in bold):

- consolidate their algebraic capability from key stage 3 and extend their understanding of algebraic simplification and manipulation to include quadratic expressions
- model situations mathematically and express the results using a range of formal mathematical representations, reflecting on how their solutions may have been affected by any modelling assumptions
- translate simple situations or procedures into algebraic expressions or formulae; derive an equation (or two simultaneous equations), solve the equation(s) and interpret the solution
- select appropriate concepts, methods and techniques to apply to unfamiliar and non-routine problems; interpret their solution in the context of the given problem.
- solve two simultaneous equations in two variables (linear/linear **{or linear/quadratic}**) algebraically;
- recognise, sketch and interpret graphs of linear functions and quadratic functions.

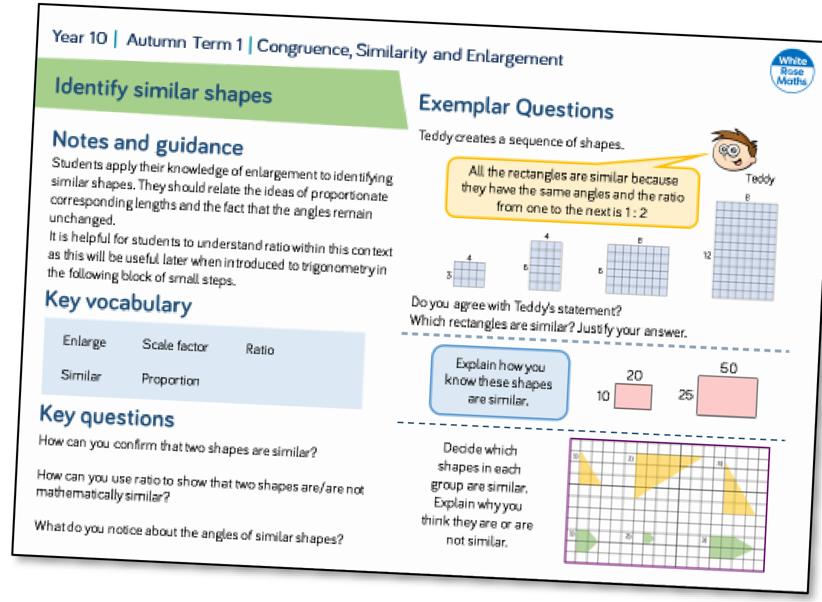
Why Small Steps?

We know that breaking the curriculum down into small manageable steps should help students to understand concepts better. Too often, we have noticed that teachers will try and cover too many concepts at once and this can lead to cognitive overload. We believe it is better to follow a "small steps" approach.

As a result, for each block of content in the scheme of learning we will provide a "small step" breakdown. ***It is not the intention that each small step should last a lesson – some will be a short step within a lesson, some will take longer than a lesson.*** We would encourage teachers to spend the appropriate amount of time on each step for their group, and to teach some of the steps alongside each other if necessary.

What We Provide

- Some ***brief guidance*** notes to help identify key teaching and learning points
- A list of ***key vocabulary*** that we would expect teachers to draw to students' attention when teaching the small step,
- A series of ***key questions*** to incorporate in lessons to aid mathematical thinking.
- A set of questions to help ***exemplify*** the small step concept that needs to be focussed on.


Year 10 | Autumn Term 1 | Congruence, Similarity and Enlargement

Identify similar shapes

Notes and guidance

Students apply their knowledge of enlargement to identifying similar shapes. They should relate the ideas of proportionate corresponding lengths and the fact that the angles remain unchanged.

It is helpful for students to understand ratio within this context as this will be useful later when introduced to trigonometry in the following block of small steps.

Key vocabulary

Enlarge, Scale factor, Ratio, Similar, Proportion

Key questions

How can you confirm that two shapes are similar?

How can you use ratio to show that two shapes are/are not mathematically similar?

What do you notice about the angles of similar shapes?

Exemplar Questions

Teddy creates a sequence of shapes.

All the rectangles are similar because they have the same angles and the ratio from one to the next is 1 : 2

Do you agree with Teddy's statement? Which rectangles are similar? Justify your answer.

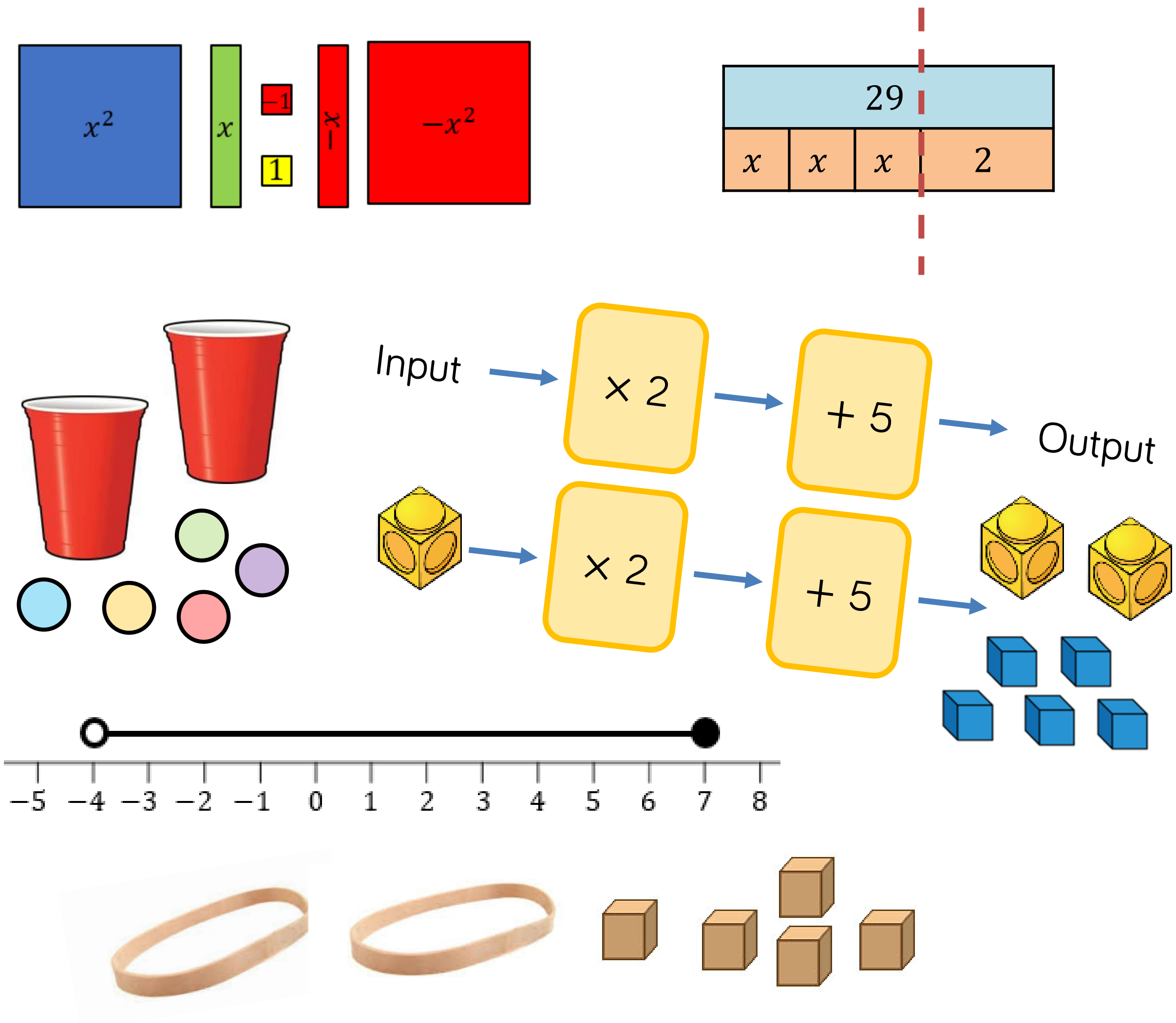
Explain how you know these shapes are similar.

Decide which shapes in each group are similar. Explain why you think they are or are not similar.

- These include reasoning and problem-solving questions that are fully integrated into the scheme of learning. Depending on the attainment of your students, you many wish to use some or all of these exemplars, which are in approximate order of difficulty. Particularly challenging questions are indicated with the symbol .
- For each block, we also provide ideas for key representations that will be useful for all students.

Some of the small steps are in **bold** and labelled with H to indicate this is Higher tier GCSE content. We would encourage teachers to use these with as many students as possible – if you feel your class can access any particular small step, then please include it in your planning. Steps that review content covered at Key Stage 3 are labelled R .

Key Representations



Here are a few ideas for how you might represent algebraic expressions and the solutions of equations and inequalities.

Cups, cubes and elastic bands lend themselves well to representing an unknown, whereas ones (from Base 10) and counters work well to represent a known number. Be careful to ensure that when representing an unknown, students use equipment that does not have an assigned value – such as Base 10 equipment and dice.

Bar models are useful to support the forming of equations and also help students to make sense of the approach to a solution. Algebra tiles are also very powerful for this and help to make sense of factorising quadratics, alternate representations are very effective in ensuring all students, including higher attaining, make sense of the mathematical structures.

Equations and Inequalities

Small Steps

- Understand the meaning of a solution
- Form and solve one-step and two-step equations R
- Form and solve one-step and two-step inequalities R
- Show solutions to inequalities on a number line
- Interpret representations on number lines as inequalities
- **Represent solutions to inequalities using set notation** H
- Draw straight line graphs R
- Find solutions to equations using straight line graphs

H Denotes Higher Tier GCSE content

R Denotes 'review step' – content should have been covered at KS3

Equations and Inequalities

Small Steps

- **Represent solutions to single inequalities on a graph** H
- **Represent solutions to multiple inequalities on a graph** H
- Form and solve equations with unknowns on both sides R
- Form and solve inequalities with unknowns on both sides
- Form and solve more complex equations and inequalities
- **Solve quadratic equations by factorisation*** (*Also Foundation tier. Higher cover now, Core will cover in Year 11) H
- **Solve quadratic inequalities in one variable** H

H Denotes Higher Tier GCSE content

R Denotes 'review step' – content should have been covered at KS3

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Understand meaning of solution

Notes and guidance

In preparation for the small steps to come, students consider what the meaning of a solution is and how they can represent this. Students consider whether a number is a solution or not by substitution and checking. They also consider how many solutions an equation could have through reasoning about different types of equations as well as why an expression would not have a solution.

Key vocabulary

Variable	Solve	Solution
Equation	Expression	

Key questions

What does the word solve mean? What connection does this have to the word solution?

Do solutions to equations have to be integers?

Can an expression have a solution?

Can an identity have a solution?

Exemplar Questions

For each of the equations, circle the correct solutions.

- $b + 8 = 25$ — $b = 17$ or $b = 33$
- $11 = 4m - 15$ — $m = -1$ or $m = 6.5$
- $10 - c = 2$ — $c = 12$ or $c = 8$
- $f + g = 10$ — $f = 7$ and $g = 3$ or $f = 11$ and $g = -1$

Is there only one solution for each of these equations? Explain how you know.

How many solutions do each of the following have? Discuss with your partner how you know.

	No solution	One solution	More than one solution
$2a + 3b = 12$			
$3c + 4c = 49$			
$3g + 4g - 5$			
$3y = 10 + y$			
$25 = t^2 + s^2$			
$p + 7 = p$			

Tom says, "in the equation $5a^2 - 3a^2 = 128$, there's only one unknown, so there's only one solution" Do you agree or disagree? Why?

Form and solve equations R

Notes and guidance

In this review step, students practice forming and solving equations. Manipulatives such as cups and counters or algebra tiles could be useful to support students as can pictorial representations such as bar models or number lines. These can support understanding of the balance method and use of inverse relationships. Use this step to revisit other topics such as angle facts, probability etc.

Key vocabulary

Equation	Solve	Inverse
Solution	Balance	

Key questions

What does solve mean?

Does it matter which order the terms in an equation are written?

What is the same and what is different about the solution to each of these scenarios?

Exemplar Questions

Solve the equations.

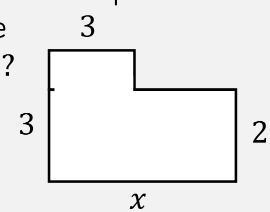
- $12g = 60$
- $5 + 3x = 44$
- $14 = \frac{t}{5} + 6$
- $12 = 60g$
- $44 + 3x = 5$
- $14 = \frac{t}{5} - 6$

Explain why each story matches with the equation $2x + 3 = 23$
What does x represent in each story?

A taxi meter starts at £3
It then costs £2 for every mile.
If the ride costs £23 altogether, how many miles is the journey?

I think of a number.
I double it and add 3
My answer is 23
What was the original number?

The area of this shape is 23 cm^2
What is the length of x?



A taxi meter starts at £3
It then costs £2 for every mile.
If the ride costs £23 altogether, how many miles is the journey?

The angles in a triangle form a linear sequence with common difference 10. If the smallest angle is $x°$, form and solve an equation to work out the angles in the triangle.
(Hint: You may use a bar model to help you).

Form and solve inequalities R

Notes and guidance

It is useful to compare and link equations and inequalities. Beware of students changing the inequality sign to an equals sign to 'make it easier' and also assuming an integer solution is needed e.g. giving the solution $x > 5.5$ as $x = 6$ or even $x > 6$. They also need to be aware that sometimes questions will ask for smallest/greatest integer so they need to pay close attention to the question.

Key vocabulary

Inequality	Solve	Inverse
Solution set	Greater/less than (or equal)	

Key questions

What's the same and what's different about solving an equation or an inequality?

How many solutions does an inequality have?

Does an inequality still hold true if you multiply/divide both sides by a negative number? Why?

Exemplar Questions

Form an equation for each of the scenarios. What's the same and what's different?

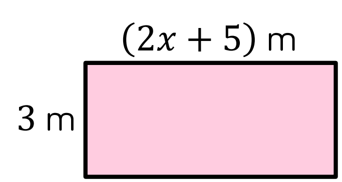
Alex is x years old. Ron is twice as old. The sum of their ages is 96

Alex is x years old. Ron is twice as old. The sum of their ages is less than 96

Alex is x years old. Ron is twice as old. The sum of their ages is at least 96

Form and solve an inequality for the following:

- I think of a number. I multiply it by 3. I then add 17. My answer is greater than 28 What is the smallest integer the number could be?
- Rosie buys 5 pens. She also buys a ruler for 70p. She pays with a five-pound note. What is the most each pen could have cost?
- The area of this rectangle is between 27 m^2 and 39 m^2 (inclusive)



What is the maximum and minimum perimeter of the rectangle?

Show solutions on a number line

Notes and guidance

It can be useful to encourage students to read the inequalities out loud to help them negotiate the meaning of the inequality symbols and how to represent them on a number line. Students should first be introduced to conventions of this topic, such as the meaning of the shading of the circle i.e. a shaded circle means the number is included, unshaded means the number is not included.

Key vocabulary

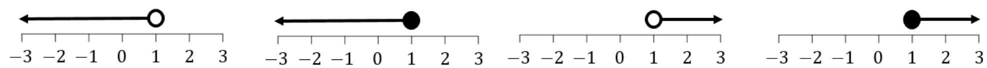
Solution set	Greater/less than (or equal to)
Inequality	Number line

Key questions

How would you read the inequality out loud?
What are the possible integer solutions for this inequality?
Is e.g. −1, 5, 0.5 a possible solution for this inequality? Why?
What does the circle mean? Which direction will the line go?

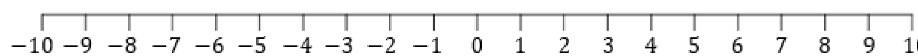
Exemplar Questions

What's the same and what's different about each of the diagrams?
Match each number line to the inequality it represents.



$x > 1$ $\quad$ $x < 1$ $\quad$ $x \geq 1$ $\quad$ $x \leq 1$

Show the solutions for these inequalities on a number line.



$x \geq 6$ $\quad$ $x < 6$ $\quad$ $-3 < x < 6$ $\quad$ $-3 \leq 3x < 6$

$x \leq 2$ $\quad$ $x + 2 \leq 2$ $\quad$ $-1 < x + 2 \leq 2$ $\quad$ $-1 \leq 3x + 2 \leq 2$

Show the possible solutions on a number line.

- I think of a number. I add 7. I then double it. My answer is less than 30
- Dexter buys 2 packs of stickers. He also buys a magazine for £4. He pays with a ten-pound note, and gets less than £3 change. What can we say about the price of a pack of stickers?

Number line representations

Notes and guidance

Now students interpret the meaning of a given number line representation and put it into an inequality format. Again, the meaning of the shading of the circle, the direction of the line and how this relates to the inequality format needs discussion. Stem sentences may be useful here to guide students. It is worth revisiting this notation regularly in starters (working both ways) to aid retention.

Key vocabulary

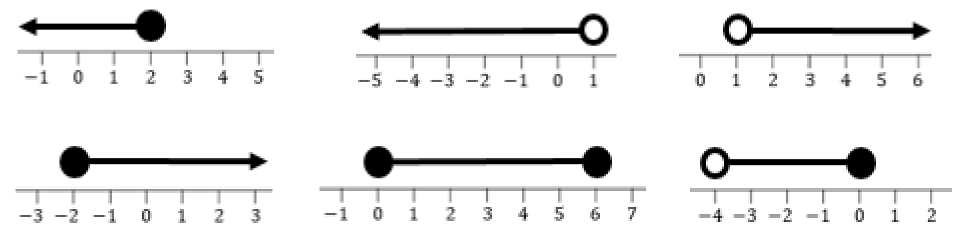
Solution set	Greater/less than (or equal to)
Inequality	Number line

Key questions

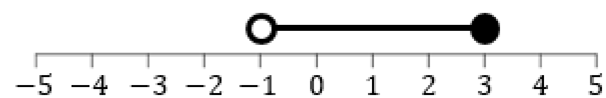
Do the solution sets contain only integers?
What's the difference in meaning if the circle is shaded or unshaded?
What direction is the line going? Are the values of the unknown smaller or greater than the number(s) shown by the circle(s)? How can you show this in an inequality?

Exemplar Questions

Write down the inequality represented by each diagram.

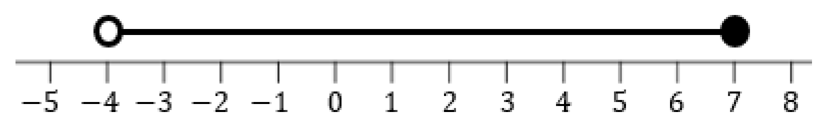


The solution for x is represented on the number line.
Are the statements below true or false? Explain how you know.



$-1 \leq x < 3$	x is greater than −1, but less than or equal to 3	x must be 0, 1, 2, or 3

The diagram shows the possible range of values for a number x



Find the single value of x if you are also given that:

- x is prime number
- $x^2 \leq 36$
- $x - 5 > -2$

Solutions using set notation

H

Notes and guidance

In this higher step, students make links between the number line representation, the verbal description and formal set notation. Students will be familiar with the term 'union' from Year 7 and afterwards, but the colon notation meaning 'such that' will be new. The key aspect of this step is flexibility, so matching activities to compare representations are particularly useful here.

Key vocabulary

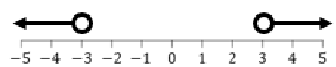
Set notation	The solution set is x such that…
Solution set	Union

Key questions

Which representation do you think is the easiest/hardest to understand?
How would you read this out loud?
What's the same and what's different about these representations?

Exemplar Questions

Match each number line with its corresponding inequality and solution given in set notation.



$\{x: -3 < x \leq 3\}$

$\{x: x > 3 \cup x < -3\}$

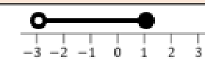
$\{x: x \leq 3\}$

$7x - 13 \leq 8$

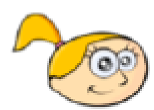
$-9 < 4x + 3 \leq 15$

$x^2 > 9$

Complete the table.

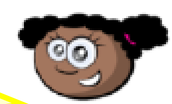
Inequality	Number line representation	Solution set
		
		
$1 \geq 10x - 7$		
$9 \leq 2x - 11 < 17$		
		$\{x: -3 \leq x < 0\}$

Eva and Whitney have both written the solution for the inequality $\frac{3x+7}{2} \geq 35$ using set notation.



Eva: $\{x: 3x + 7 \geq 70\}$

Whitney: $\{x: x \geq 21\}$



Are they both correct? Explain why.

Draw straight line graphs R

Notes and guidance

This review step reminds students how to draw linear graphs, making connections between the representations as a graph, an equation, a table of values and a set of coordinates. Students should be encouraged to look for errors in their table of values if their points do not form the expected straight line. Both calculator and non-calculator methods of dealing with negative numbers should be explored.

Key vocabulary

Gradient	Positive/Negative	Linear
y-intercept	Coordinate	Plot

Key questions

How do you decide what values of x to choose for a table of values?

What does the gradient of a graph tell you?

Why is it helpful to have the equation in the format $y = mx + c$ in order to plot a linear graph?

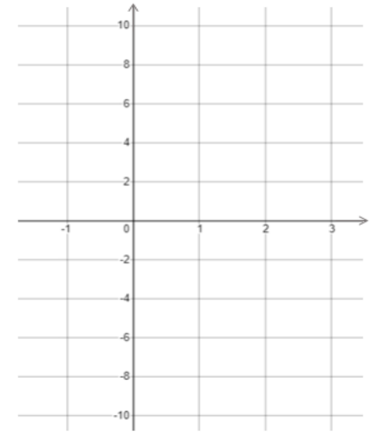
How many solutions does the equation of a straight line have?

Exemplar Questions

Complete the table of values for $y = 4x - 3$

On the grid, draw the graph of $y = 4x - 3$

x	-1	0	1	2	3
y					



Plot and label the following sets of graphs. What's the same and what's different?

$y = 5x$
$y = 3x$
$y = -3x$
$y = \frac{1}{2}x$

$y = 5x + 4$
$y = 3x + 4$
$y = -3x + 4$
$y = -3x - 4$

$y = 3(x + 2)$
$y = 3(x - 2)$
$y = \frac{(x + 2)}{2}$

Mo and Annie explain how they plotted the line $y = 3x + 2$
Draw the graph two times using the methods Mo and Annie describe.
Which do you prefer?

First, I plotted (0, 2)
From this point, for every one I moved across on the x-axis, I moved up 3 on the y-axis.
I then completed the line.

I completed a table of values for x for the range -2 to 2
I then plotted each coordinate pair and completed the line.

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Find solutions graphically

Notes and guidance

Here students learn the connection between solving algebraically and solving graphically. It can be useful to draw attention to the fact that for a linear equation there will only be one point where the graphs meet and the x value corresponds to the solution of the linear equation. This is useful conceptual understanding that will help students later with solving simultaneous equations.

Key vocabulary

Set equal	Solution	Intersect
Solve graphically	Solve algebraically	

Key questions

Why don't we need to draw tables of values for graphs like $y = 3$ and $x = -2$?

How do we know which graphs to draw to solve e.g. $5x - 2 = 9$?

Is it always possible to solve two sets of equations graphically? Is a graphical method always going to be useful?

Exemplar Questions

Draw a set of coordinate axes from -6 to 6 in both directions.
Show these straight lines on your grid.

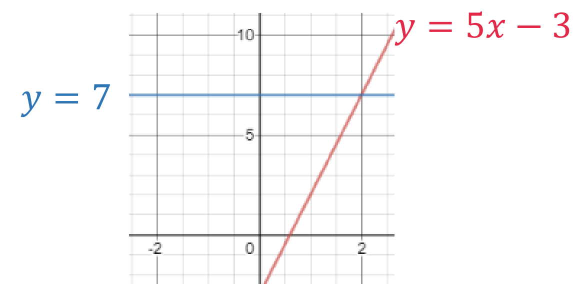
$x = 3$ $\quad$ $y = 1$ $\quad$ $x = -2$ $\quad$ $y = -4$

Where do the following pairs of lines meet?

- $x = 3$ and $y = -4$
- $x = -2$ and $y = -4$
- $y = 1$ and $x = -2$
- Explain why $x = 3$ and $x = -2$ never meet

What's the same and what's different about these representations?

$$5x - 3 = 7$$
$$+3 \qquad +3$$
$$5x = 10$$
$$\div 5 \qquad \div 5$$
$$x = 2$$



Which of these graphs would you draw to solve $\frac{1}{2}x + 5 = 3$?

$x = 3$ $\quad$ $y = 3$ $\quad$ $y = \frac{1}{2}x + 5$ $\quad$ $x = \frac{1}{2}y + 5$

Find where $y = \frac{1}{2}x + 3$ meets $y = 4$, $y = -1$ and $y = -1.5$

Write down the equations you can solve using your answers.

Single inequalities on a graph H

Notes and guidance

Students need to know the convention that e.g. $x \leq 3$ is represented by shading to the left of the solid line $x = 3$ whilst $x < 3$ the line would be dashed, linking to the shading of circles in number lines representing inequalities. Testing a single point above or below the line $y = 3x - 1$ is useful to decide where to shade e.g. $y > 3x - 1$; using the origin is a good strategy.

Key vocabulary

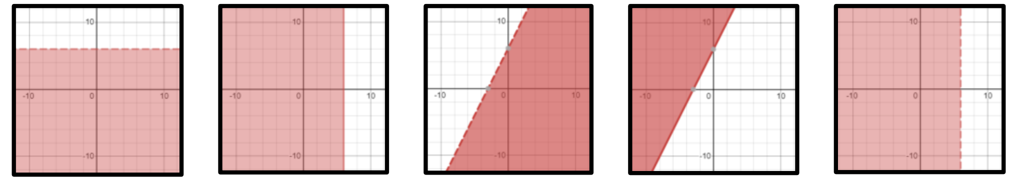
Inequality	Satisfy	Region
Dashed line	Solid line	Test point

Key questions

What's the same and what's different about the graphs?
What is the significance of the dashed line and solid line when looking at regions of inequalities?
Does the point e.g. (4, 2) satisfy this inequality? How can we find out?
Why is (0, 0) a good point to use as a test point? Why would (0, 0) not be suitable for regions like $y > -2x$?

Exemplar Questions

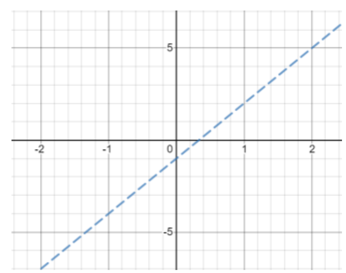
Match the inequalities to their graphical representation

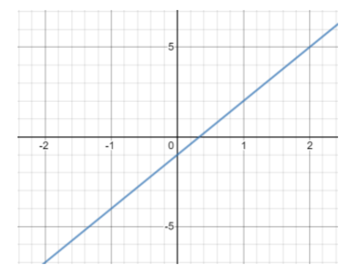


$x \leq 6$ $x < 6$ $y < 2x + 6$ $y < 6$ $y \geq 2x + 6$

The line $y = 3x - 1$ is shown in both of the graphs below.

- Choose the appropriate graph and shade the region that satisfies the inequality $y < 3x - 1$
- Choose the appropriate graph and shade the region that satisfies the inequality $y \geq 3x - 1$





For each inequality draw a pair of coordinate axes going from -4 to 4 in both directions, and shade the region indicated.

- $x < 3$
- $y \geq 0$
- $y \geq \frac{1}{2}x + 1$
- $x + 3y < 9$

For which of the inequalities is (3, 0) a solution?
How can you show this graphically? Algebraically?

Multiple inequalities on graph H

Notes and guidance

Students extend their knowledge of representing inequalities on a graph to being able to shade regions defined by multiple inequalities. Students should practice both working from the graphs and writing in the inequalities and starting from the inequalities, shading the regions that are satisfied by the inequalities. Again comparing test points in/out of the regions is useful.

Key vocabulary

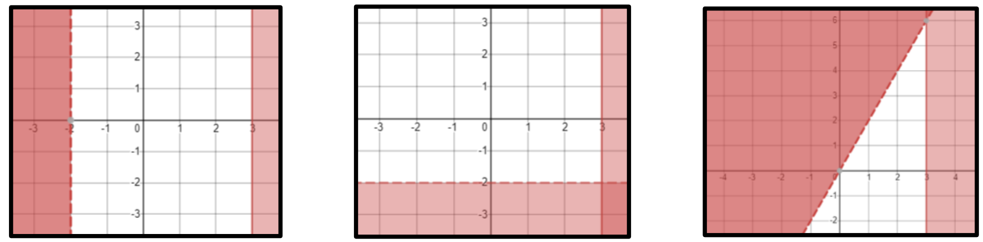
Inequality	Satisfy	Region
Dashed line	Solid line	Test point

Key questions

How can you show that the point e.g. (4, 2) satisfies each of the inequalities?
How do you decide which side of a line to shade in and which side not to shade in?
How do you show that the points on a line are included in/excluded from a solution set?

Exemplar Questions

Which inequalities satisfy the **unshaded** regions?
Give your answers in set notation.

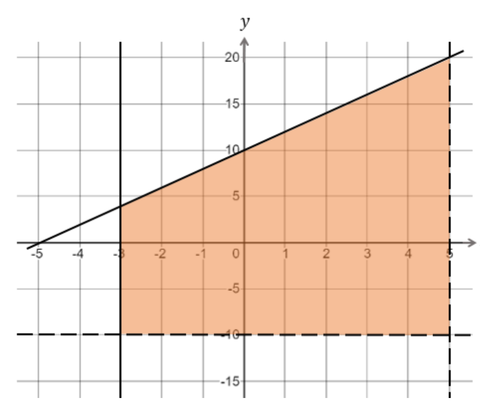


On a pair of coordinate axes going from −5 to 5 for both directions, shade the region that is satisfied by each pair or group of inequalities.

$x \geq -1$ and $y < 0$	$y \geq -2$ and $y < 2x$
$x < 0$ and $y + x > -1$	$y \geq -1, x < 2$ and $y > 2x - 1$

Find the equations of the lines that enclose the trapezium.

- Write the inequalities that are satisfied by this region
- How many solutions are there to the set of inequalities where x and y are both integers?
- Work out the area of the trapezium



Equations: unknown both sides R

Notes and guidance

Students have met equations of this form at key stage 3, so teachers will need to decide how much consolidation or practice is needed when revisiting this important topic. The use of concrete materials such as counters and cups as well as the pictorial support of bar models can be used to aid student understanding. As well as practising solving, discussion on how to form the equations is key.

Key vocabulary

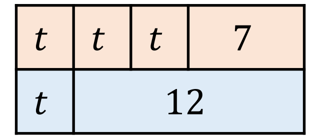
Balance	Is equal to	Value
Solution	Unknown	Satisfy

Key questions

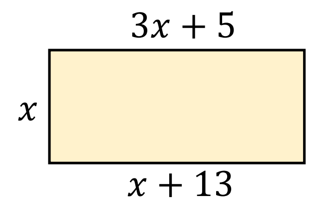
How many values of x will satisfy this equation? Why?
Explain why $8x + 3 = 8x + 9$ has no solution.
Will the solutions to $4x + 8 = 9x + 5$ be the same as the solutions to $9x + 5 = 4x + 8$?
What about $6x + 4 = 10$ and $3x + 2 = 5$

Exemplar Questions

What equation is represented by the bar model?
Solve the equation to find the value of t

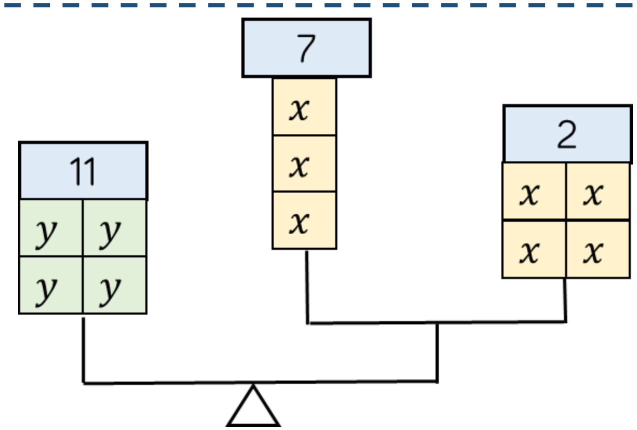


Find the perimeter and area of the rectangle.



I think of a number multiply it by 5 and subtract 12
My answer is 18 greater than my original number.
Form and solve an equation to find my original number.
Make up your own multi-step number puzzles and challenge a partner

The diagram shows a balance on another balance.
Work out the values of x and y.



Inequalities: unknown both sides

Notes and guidance

This step provides consolidation of the techniques covered in the previous step and the number line notation met earlier in this block. Again teachers will need to be vigilant for students changing or omitting inequality signs.

Higher tier students should be encouraged to also give their answers in set notation to further practice this skill.

Key vocabulary

Less/greater than	Or equal to	Solution set
Linear	Inequality	Number line

Key questions

Explain the difference between an inequality and an equation.
What is the difference between ≤ and <?
Explain the difference between $x < 7$ and $7 > x$
Will the solution set of $4x + 8 > 9x + 5$ be the same as the solution set of $9x + 5 > 4x + 8$?

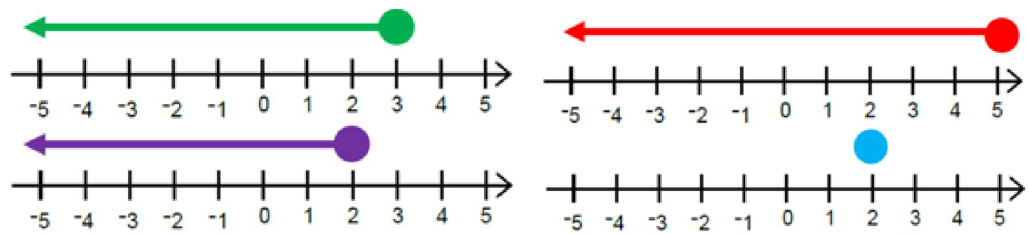
Exemplar Questions

Solve these linear inequalities.

- $3y + 12 < 4$
- $3y + 12 \leq -4$
- $3y + 12 < y - 4$
- $3y + 12 < y + 4$
- $3y - 12 \leq y - 4$
- $3y - 12 \geq y - 4$

What is the same and what is different?

Which number line represents the solution to $9x - 4 \leq 7x + 2$?

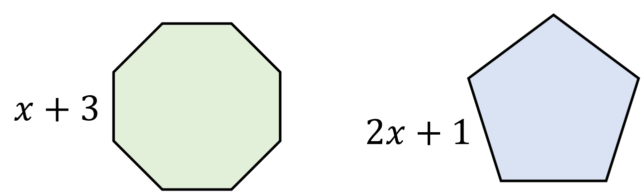


Draw a number line to show the solutions to the inequalities

$\frac{7a-5}{2} > 2a - 1$ $\qquad$ $2b + 7 \geq \frac{b}{2} + 1$

The perimeter of the regular octagon is less than the perimeter of the regular pentagon.

- Show this information as an inequality in terms of x
- Find the smallest possible integer value of x



Complex equations & inequalities

Notes and guidance

Students will now solve equations and inequalities where brackets may be present on one or both sides and/or more challenging contexts. The aim is to develop fluency within wider mathematics and not purely algebraic settings. Students should be exposed to different ways of answering the same question, such as multiplying the brackets out first or dividing.

Key vocabulary

Less than	Greater than	Solution(s)
Linear	Balanced	Inequality

Key questions

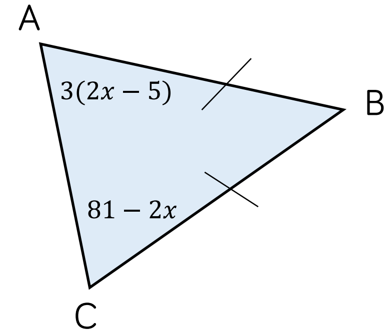
Compare (e.g.) $\frac{4x+10}{3} > 2x + 5$ and $\frac{4x}{3} + 10 > 2x + 5$
What is different about how you solve these?

Do you always need to expand brackets when they occur in an equation?

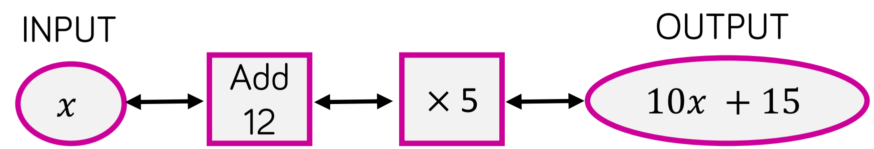
Explain the order of the steps you would take to solve…

Exemplar Questions

Calculate the size of angle ABC



Calculate the value of x.



List the integer values that satisfy both $4(x + 5) \leq 24$ and $3(3x + 1) > 7x - 9$

The angles in a triangle are $x + 50, x + 20$ and $x - 10$.
Show that the triangle is right-angled.

The solutions to the equations form a linear sequence.
Write an equation whose solution is the 4th term of the sequence.
First term: $3x + 5 = 4(x - 1.5)$
Second term: $3(2x + 1) = 5(x + 2)$
Third term: $2(x - 2) = 5 - x$

Quadratics using factorisation H

Notes and guidance

Higher tier students will have met factorisation of quadratic expressions in Year 9. Using algebra tiles and linking to area models/factors of numbers provides a solid base for making sense of factorisation. Students should also consider expressions that cannot be factorised. Making links to graphical representation of the equations is useful here and will support the next step dealing with quadratic inequalities.

Key vocabulary

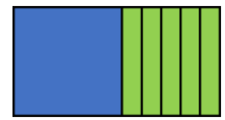
Quadratic	Roots	Solution(s)
Factorise	Brackets	

Key questions

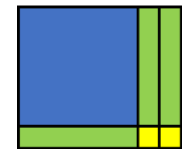
Find some solutions to $ab = 12$ and $ab = 0$
What's the same and what's different?
How many rectangles can you make using the tiles? (e.g. 1 of x^2, 5 of x and 6 ones). Are they the same or different?
How can you tell if an equation is quadratic or linear just by looking at it? Can you think of other types of equation?
Do all quadratic equations have two solutions?

Exemplar Questions

Explain why these diagrams of algebra tiles show the given factorisations.



$x^2 + 5x \equiv x(x+5)$



$x^2 + 3x + 2 \equiv (x+1)(x+2)$

Using algebra tiles factorise the expressions, then explain why $x^2 + 3x + 5$ cannot be factorised.

$x^2 + 5x + 6$ | $x^2 + 7x + 6$ | $x^2 + 5x + 4$ | $x^2 + 4x + 4$

Which of these equations have only one solution, exactly two solutions or more than two solutions?

$2x = 0$
$x^2 = 0$
$xy = 0$
$x(x-1) = 0$
$x(x+1) = 0$
$(x+1)(x-1) = 0$

$(x+2)(x-1) = 0$
$(x+2)(x+1) = 0$
$(2-x)(1-x) = 0$
$(2-x)(1+x) = 0$
$2(2-x)(1+x) = 0$

Spot the errors in this solution.

$x^2 + 2x = 8$
$x(x+2) = 8$
Either $x = 8$ or $x + 2 = 8$
$x = 8$ or $x = 6$

Rearrange and solve the equations.

$x^2 + 2x = 8$ | $x^2 - 2x = 8$
$x^2 - 2x = 15$ | $x^2 + 2x = 3$

Quadratic inequalities

H

Notes and guidance

Using graphing software to look at multiple graphs and identifying regions is an excellent introduction to this topic. Students need to be confident in identifying which region is above or below the x axis and how this affects the solution to a quadratic inequality. Students should be encouraged to always find the critical values and draw a sketch (rather than plot the whole graph) to identify the region(s) needed.

Key vocabulary

Roots	Solutions	Intercept
x-axis	Factorise	Sketch

Key questions

How does $\leq$ or $\geq$ change the solution?

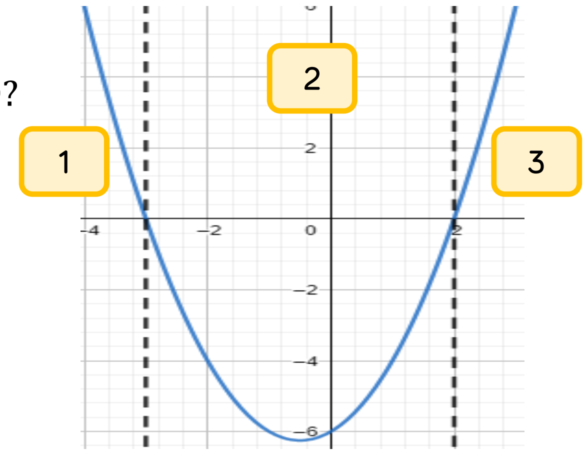
How does the solution set of …> 0 differ from the solution set of …. ≥ 0?

How do we know whether to look above or below the x-axis?

What is the first step we need to take to solve a quadratic inequality.?

Exemplar Questions

Which of the region(s) 1, 2 or 3 on the graph represent(s) the solution of $(x-2)(x+3) > 0$?



Solve the inequality $(x-2)(x+3) > 0$, showing your answer on a number line, in set notation and as a pair of inequalities.

Sketch the graph of $y = x^2 + 4x - 5$, showing where the curve meets the axes. Use your graph to solve the inequalities.

$x^2 + 4x - 5 > 0$ | $x^2 + 4x - 5 \leq 0$

$x^2 + 4x \geq 5$ | $x^2 + 4x < 5$ | $x^2 > 5 - 4x$

Find the set of values that satisfies both $x^2 - 3x - 10 < 0$ and $7x + 5 > 8 + 4x$, showing your answer on a number line.

Explain why $x^2 + 4 < 0$ has no solutions.