Manipulating Expressions

Year 10

#MathsEveryoneCan

White Rose Maths

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	Week 1	Week 2	Week 3	Week 4	Week 5	Week 6	Week 7	Week 8	Week 9	Week 10	Week 11	Week 12
Autumn	**Similarity**						**Developing Algebra**					
	Congruence, similarity and enlargement			Trigonometry			Representing solutions of equations and inequalities			Simultaneous equations		
Spring	**Geometry**						**Proportions and Proportional Change**					
	Angles & bearings		Working with circles		Vectors		Ratios & fractions		Percentages and Interest		Probability	
Summer	**Delving into data**				**Using number**						**Expressions**	
	Collecting, representing and interpreting data				Non-calculator methods		Types of number and sequences		Indices and Roots		Manipulating expressions	

Summer 2: Using Number/Expressions

Weeks 1 and 2: Types of Number and Sequences

This block again mainly revises KS3 content, reviewing prime factorisation and associated number content such as HCF and LCM. Sequences is extended for Higher Tier to include surds and finding the formula for a quadratic sequence.

National curriculum content covered:

- consolidating subject content from key stage 3:
  - factors, multiples, primes, HCF and LCM
  - describe and continue sequences
- recognise and use sequences of triangular, simple arithmetic progressions, Fibonacci type sequences, quadratic sequences, and simple geometric progressions (r^n where n is an integer, and r is a positive rational number **{or a surd}) {and other sequences}**
- deduce expressions to calculate the nth term of linear **{and quadratic}** sequence

Weeks 3 and 4: Indices and roots

This block consolidates the previous two blocks focusing on understanding powers generally, and in particular in standard form. Negative and fractional indices are explored in detail. Again, much of this content will be familiar from KS3, particularly for Higher tier students, so this consolidation material may be covered in less than two weeks allowing more time for general non-calculator and problem-solving practice. To consolidate the index laws, these can be revisited in the next block when simplifying algebraic expressions.

National curriculum content covered:

- recognise and use sequences of square and cube numbers
- **{estimate powers and roots of any given positive number}**
- calculate with roots, and with integer **{and fractional}** indices
- calculate with numbers in standard form $A \times 10^n$, where $1 \leq A < 10$ and n is an integer
- simplifying expressions involving sums, products and powers, including the laws of indices

Weeks 5 and 6: Manipulating expressions

This final block of year 10 builds on the Autumn term learning of equations and inequalities, providing revision and reinforcement for Foundation tier students and an introduction to algebraic fractions for those following the Higher tier. This also allows all students to revise fraction arithmetic to keep their skills sharp. Algebraic argument and proof are considered, starting with identities and moving on to consider generalised number.

National curriculum content covered:

- simplify and manipulate algebraic expressions (including those involving surds **{and algebraic fractions}** by factorising quadratic expressions of the form $x^2 + bx + c$
- know the difference between an equation and an identity
- argue mathematically to show algebraic expressions are equivalent, and use algebra to support and construct arguments **{and proofs}**

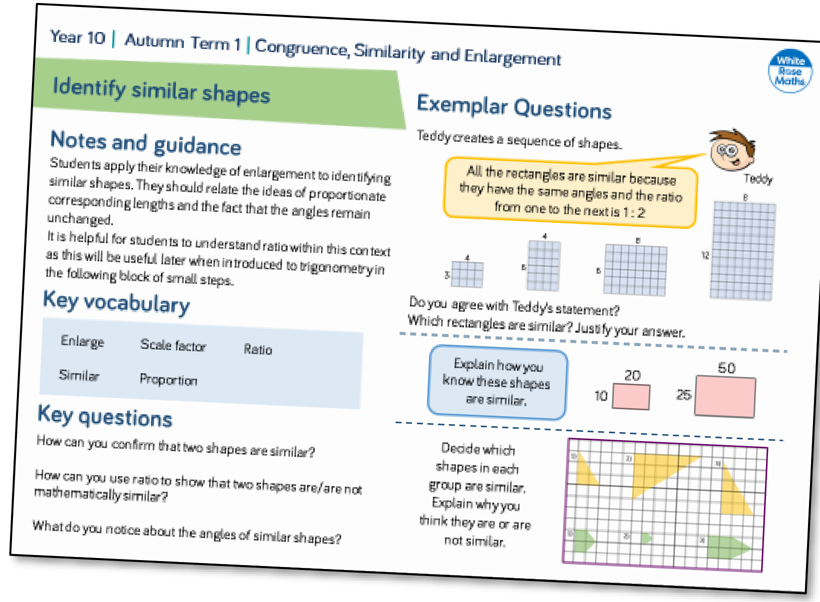
Why Small Steps?

We know that breaking the curriculum down into small manageable steps should help students to understand concepts better. Too often, we have noticed that teachers will try and cover too many concepts at once and this can lead to cognitive overload. We believe it is better to follow a "small steps" approach.

As a result, for each block of content in the scheme of learning we will provide a "small step" breakdown. ***It is not the intention that each small step should last a lesson – some will be a short step within a lesson, some will take longer than a lesson.*** We would encourage teachers to spend the appropriate amount of time on each step for their group, and to teach some of the steps alongside each other if necessary.

What We Provide

- Some ***brief guidance*** notes to help identify key teaching and learning points.
- A list of ***key vocabulary*** that we would expect teachers to draw to students' attention when teaching the small step.
- A series of ***key questions*** to incorporate in lessons to aid mathematical thinking.
- A set of questions to help ***exemplify*** the small step concept that needs to be focussed on.


Year 10 | Autumn Term 1 | Congruence, Similarity and Enlargement

Identify similar shapes

Notes and guidance

Students apply their knowledge of enlargement to identifying similar shapes. They should relate the ideas of proportionate corresponding lengths and the fact that the angles remain unchanged.

It is helpful for students to understand ratio within this context as this will be useful later when introduced to trigonometry in the following block of small steps.

Key vocabulary

Enlarge, Scale factor, Ratio, Similar, Proportion

Key questions

How can you confirm that two shapes are similar?

How can you use ratio to show that two shapes are/are not mathematically similar?

What do you notice about the angles of similar shapes?

Exemplar Questions

Teddy creates a sequence of shapes.

All the rectangles are similar because they have the same angles and the ratio from one to the next is 1 : 2

Do you agree with Teddy's statement? Which rectangles are similar? Justify your answer.

Explain how you know these shapes are similar.

Decide which shapes in each group are similar. Explain why you think they are or are not similar.

- These include reasoning and problem-solving questions that are fully integrated into the scheme of learning. Depending on the attainment of your students, you may wish to use some or all of these exemplars, which are in approximate order of difficulty. Particularly challenging questions are indicated with the symbol 💡.
- For each block, we also provide ideas for key representations that will be useful for all students.

Some of the small steps are in **bold** and labelled with H to indicate this is Higher tier GCSE content. We would encourage teachers to use these with as many students as possible – if you feel your class can access any particular small step, then please include it in your planning. Steps that review content covered at Key Stage 3 are labelled R.

Manipulating expressions

Small Steps

- Simplify algebraic expressions (R)
- Use identities
- **Add and subtract simple algebraic fractions** (H)
- **Add and subtract complex algebraic fractions** (H)
- **Multiply and divide simple algebraic fractions** (H)
- **Multiply and divide complex algebraic fractions** (H)
- Form and solve equations and inequalities with fractions
- **Solve equations with algebraic fractions** (H)

(H) denotes Higher Tier GCSE content

(R) denotes 'review step' – content should have been covered at KS3

Manipulating expressions

Small Steps

- Represent numbers algebraically
- Algebraic arguments and proof

H denotes Higher Tier GCSE content

R denotes 'review step' – content should have been covered at KS3

Simplify expressions

R

Notes and guidance

Students should be secure in simplifying expressions using addition, subtraction, multiplication and division. As well as revising like and unlike simple terms, encourage students to see that e.g. $2(a+b)+3(a+b)$ can be simplified without expanding the brackets. You can also revisit the rules of indices from the previous block. Special care is needed with negative signs when finding the difference between expressions e.g. $4a-3-(2a-5)$.

Key vocabulary

Expression	Term	Simplify
Coefficient	Power	Like/unlike

Key questions

True or false – "When you add/subtract like terms, you only need to add/subtract their coefficients?"
How can you tell if two terms are like or unlike?
Does the order of the letters in a term matter?
What's different about simplifying when you're multiplying/dividing rather than adding/subtracting?

Exemplar Questions

Which of these expressions can be simplified?

$2+3a$	$2a+3a$	$2ab+3ab$	$2ab+3ba$
$2a+3a^2$	$2a^2+3a^3$	$2(a+b)+3(a+b)$	

Write those that can be simplified in their simplest form.
How would your answers change if all the $+$ signs were replaced by $-$?

What's the same and what's different about simplifying these expressions?

$p^5 \times p^3$	$p^5 \div p^3$	$p^5 + p^5$	$p^5 + p^3$

How would your approach change if the terms had coefficients greater than 1?

Dora has y sweets.
Ron has 12 more sweets than Dora.
Whitney has three times as many sweets as Dora.
Amir has twice as many sweets as Ron.

- How many expressions in y can you form using this information?
- Which expressions are definitely greater than/less than the other expressions?
- Form an expression for the total number of sweets the children have between them. How many ways can you write this expression?

Use identities

Notes and guidance

This step builds on earlier learning ensuring students understand the difference between equality and equivalence. Here students develop their knowledge to appreciate that in an identity the coefficients of each variable can be compared to find missing values i.e. if $ax + by \equiv cx + dy$ then $a = c$ and $b = d$. Students need to be confident with expanding single brackets. This is developed further in Year 11 “Expanding and Factorising”.

Key vocabulary

Expression	Variable	Simplify
Coefficient	Identity	Equivalent

Key questions

What’s the difference between an identity and an equation?

Give me three different expressions that are always identical in value to e.g. $6x - 8y$

If two expressions are identical what can you say about the coefficients of the variables?

Exemplar Questions

Which of these statements are true and which are false?

$2x + x \equiv 3x$ | $2x - x \equiv 2$ | $2(x + 3y) \equiv 2x + 23\,y$

Correct the statements that are false.

Which expressions are equivalent and which are not? Why?

$a \div 2$ | $\frac{a}{2}$ | $\frac{2}{a}$ | $\frac{1}{2}a$ | $2 \div a$

$6y + 18 \equiv 6(y + 3)$

$6y + 18 = 4y + 3$

$6y + 18 = 6y + 3$

Does each card show an identity or an equation?

Can the equations be solved?

Why or why not?

Identity 1 $3x + 8 + 4x + a \equiv bx + 12$

Identity 2 $3(x + 5) + 4(2x + 3) \equiv cx + d$

Identity 3 $4(ex + 2) + 6(x + f) \equiv 2(9x + 19)$

Use the identities to find the values of a, b, c, d, e and f.

Correct the mistakes in these identities.

$5(b + 2) \equiv 5b + 2$ ✗

$6c + 3 + 4c \equiv 13c$ ✗

+/− simple algebraic fractions H

Notes and guidance

Students may need a reminder of addition and subtraction of numerical fractions before engaging with this step. They then look at fractions with algebraic numerators and or denominators, comparing the results to those of the familiar numerical additions and subtractions. It is also worth exploring simplification of algebraic fractions within this step so that answers to additions and subtractions can be given in simplest form.

Key vocabulary

Numerator	Denominator	LCM
Sum	Difference	Simplify

Key questions

How do you find the lowest common multiple of two numbers?

How do you find the lowest common multiple of two algebraic expressions?

How can you tell if an algebraic fraction is in its simplest form?

Exemplar Questions

What's the same and what's different about adding these pairs of fractions?

$\frac{2}{7}+\frac{3}{7}$ $\quad\frac{a}{7}+\frac{b}{7}$ $\quad\frac{x}{7}+\frac{x}{7}$ $\quad\frac{x}{7}+\frac{2x}{7}$

Find the sums and differences, giving your answers as single fractions in their simplest form.

$\frac{5}{a}+\frac{3}{a}$ $\quad\frac{5}{a}-\frac{3}{a}$ $\quad\frac{x}{a}+\frac{y}{a}$ $\quad\frac{5x}{a}-\frac{2x}{a}$

Which simplifications are correct and which are incorrect? Why?

$\frac{1\cancel{0}}{3\cancel{0}}=\frac{1}{3}$ $\quad\frac{3\cancel{2}}{5\cancel{2}}=\frac{3}{5}$ $\quad\frac{3\cancel{b}}{4\cancel{b}}=\frac{3}{4}$ $\quad\frac{\cancel{c}^8}{\cancel{c}^3}=\frac{8}{3}$

$\frac{\overset{2}{\cancel{6}}p+q}{\cancel{3}}=2p+q$ $\quad\frac{\cancel{a}+b}{\cancel{a}+c}=\frac{b}{c}$

State the common denominator you would use to complete these additions and subtractions, and find the answers.

$\frac{1}{4}+\frac{3}{5}$ $\quad\frac{1}{4}+\frac{3}{8}$ $\quad\frac{1}{a}+\frac{3}{2a}$ $\quad\frac{4}{a}-\frac{1}{ab}$

+/− complex algebraic fractions H

Notes and guidance

Students build on the learning in the previous step to explore fractions with numerators and denominators involving more than a single term. They need to be confident with single bracket expansion and dealing with negative numbers, so a starter activity on these topics may be useful. Encourage students to check answers using substitution, discussing suitable values that could be used.

Key vocabulary

Numerator	Denominator	LCM
Sum	Difference	Simplify

Key questions

When are brackets important when finding the lowest common denominator of a set of algebraic fractions?

When are brackets useful on the numerators of algebraic fractions?

How do you subtract a pair of expressions? Why do you need be careful with − signs?

Exemplar Questions

Express each of the following as a single algebraic fraction.

$\frac{a}{3}+\frac{a}{5}$ $\quad$ $\frac{a}{3}-\frac{a}{5}$ $\quad$ $\frac{a+4}{3}+\frac{a+2}{5}$ $\quad$ $\frac{a+4}{3}-\frac{a+2}{5}$

$\frac{a-4}{3}+\frac{a-2}{5}$ $\quad$ $\frac{a-4}{3}-\frac{a+2}{5}$ $\quad$ $\frac{a-4}{3}-\frac{a-2}{5}$

Check your answers by substituting suitable values of a.

- Show that $\frac{1}{x}+\frac{1}{y}=\frac{y+x}{xy}$
- Show that $\frac{1}{x}+\frac{1}{y+1}=\frac{x+y+1}{x(y+1)}$
- Find $\frac{1}{x}+\frac{1}{x+1}$
- Find $\frac{3}{x}+\frac{5}{x+1}$

Dexter is working out $\frac{3}{x+2}-\frac{2}{x+1}$

He writes

$$\frac{3}{x+2}-\frac{2}{x+1}=\frac{3x+1-2x+2}{(x+2)(x+1)}=\frac{x+3}{(x+2)(x+1)}$$

- Identify one thing that Dexter has done well and the mistakes that Dexter has made.
- Show that the correct answer is $\frac{x-1}{(x+2)(x+1)}$

×/÷ simple algebraic fractions

H

Notes and guidance

Students may need to practise multiplication and division of numerical fractions first. For multiplication, students firstly explore the product of a fraction by integer with one aspect algebraic before looking at more general fractions. Division is approached in the same way. In this step numerators and denominators are kept to single terms with fractions of the form $\frac{a}{x+b}$ and $\frac{x+a}{b}$ looked at in the next step.

Key vocabulary

Numerator	Denominator	Invert
Reciprocal	Product	Quotient

Key questions

If you multiply a fraction by an integer, does the numerator or denominator change?

What's the difference between the way you approach multiplying and dividing a pair fractions?

How do you find the reciprocal of a fraction?

Exemplar Questions

Find the products.

$\frac{2}{7} \times 3$ $\quad$ $\frac{a}{7} \times 3$ $\quad$ $\frac{2}{7} \times a$ $\quad$ $\frac{2}{a} \times 3$

What's the same and what's different?

- Explain why $\frac{1}{5} \div 2 = \frac{1}{10}$
- Work out $\frac{1}{5} \div 3$, $\frac{1}{5} \div 4$ and $\frac{1}{5} \div 5$
- Which expression is the correct answer to $\frac{1}{5} \div x$? $\quad \frac{x}{5}$ $\quad \frac{5}{x}$ $\quad \frac{1}{5x}$
- Verify your answer by checking for $x = 2, 3, 4$ and 5

Mo says,

To find the product of any pair of fractions you multiply the numerators and multiply the denominators.

Use Mo's method to work out

- $\frac{2}{3} \times \frac{1}{5}$
- $\frac{2}{3} \times \frac{a}{5}$
- $\frac{2}{3} \times \frac{1}{a}$
- $\frac{a}{3} \times \frac{1}{5}$
- $\frac{a}{3} \times \frac{1}{b}$

What's different about finding the quotient of two fractions?

Work out

- $\frac{2}{3} \div \frac{1}{5}$
- $\frac{2}{3} \div \frac{a}{5}$
- $\frac{2}{3} \div \frac{1}{a}$
- $\frac{a}{3} \div \frac{1}{5}$
- $\frac{a}{3} \div \frac{1}{b}$

Find as many pairs of fractions as you can if you know

- their product is $\frac{5x}{6y}$
- their quotient is $\frac{5x}{6y}$

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×/÷ complex algebraic fractions H

Notes and guidance

Complexity is increased in this step through using more algebraic terms and expressions with more than one term. You might also wish to discuss the use of 'cancelling' when multiplying and dividing both numerical and algebraic fractions. If students have completed the higher steps on quadratics in the Autumn term algebra block, you could take this step even further, or leave the most complex multiplications and divisions until Year 11.

Key vocabulary

Numerator	Denominator	Cancel
Reciprocal	Factor	Factorise

Key questions

What does it mean to 'cancel' when multiplying and dividing fractions?

Why does it help to look for factors of the numerator and denominator?

What's the difference between the way you approach multiplying and dividing fractions?

Exemplar Questions

Compare these two methods of working out $\frac{5}{x} \div \frac{3}{2x}$

$$\frac{5}{x} \div \frac{3}{2x} = \frac{5}{x} \times \frac{2x}{3} = \frac{10\cancel{x}}{3\cancel{x}} = \frac{10}{3}$$

$$\frac{5}{x} \div \frac{3}{2x} = \frac{5}{\cancel{x}} \times \frac{2\cancel{x}}{3} = \frac{10}{3}$$

Use your preferred method to express these as single fractions in their simplest form.

- $\frac{3ab}{5} \times \frac{7}{a}$
- $\frac{3ab}{5} \div \frac{b}{10}$
- $\frac{3ab}{5} \div \frac{a^2}{b}$
- $\frac{3ab}{5} \times \frac{b}{6a}$
- $\frac{3ab}{5} \div \frac{9b}{20a}$

Work these out giving your answers as simply as possible.

$\frac{t}{3} \times \frac{t+1}{4}$	$\frac{t}{3} \times \frac{4}{t+1}$	$\frac{t-5}{3} \times \frac{4}{t}$	$\frac{2}{t} \times \frac{4}{t+3}$
$\frac{t}{3} \div \frac{t}{6}$	$\frac{t}{4} \div \frac{t+2}{5}$	$\frac{t+5}{3} \div \frac{6}{t}$	$\frac{6t}{5} \div \frac{8t}{t+3}$

Dora is working out $\frac{y+4}{2} \div \frac{2y+8}{3}$

She writes $\frac{y+4}{2} \div \frac{2y+8}{3} = \frac{y+4}{2} \times \frac{3}{2y+8} = \frac{3y+12}{4y+16}$

She checks by substituting several values of y and finds the answer is $\frac{3}{4}$ every time.

Show that Dora's answer simplifies to $\frac{3}{4}$

Is there another way Dora could have got to the answer?

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Equations and inequalities with fractions

Notes and guidance

This step provides a good opportunity to revise students' learning of equations and inequalities from the Autumn term. A common error is to multiply only some of the terms when trying to eliminate fractions, so students should be encouraged to check their answers by substituting back into the original equation or inequality. Students need to be confident with the material in this step before proceeding to equations with fractions with algebraic denominators.

Key vocabulary

Equation	Inequality	Solve
Solution	Solution set	Strict

Key questions

What do we mean by a "strict" inequality?
How can you tell if a number line represents the answer to a strict inequality?
Is it easier to simplify the fractions or multiply through so all the terms have integer coefficients?

Exemplar Questions

What's the same and what's different about solving these equations?

$2x + 7 = 19$	$\frac{x}{2} + 7 = 19$	$\frac{x + 7}{2} = 19$

Brett thinks of a number.
Four less than a third of Brett's number is greater than 13
Form and solve an inequality to show that Brett's number must be greater than 50

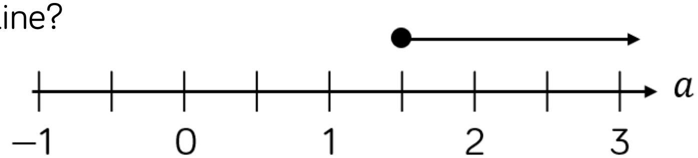
Which of the approaches are correct to solve the equation $\frac{2}{3}y = 15$?

Multiply both sides by 3 and then divide both sides by 2	Divide both sides by 2 and then multiply both sides by 3	Divide both sides by $\frac{2}{3}$

Represent the solution to each inequality on a number line.

$\frac{3x + 7}{2} < 17$	$\frac{1}{3}x + 5 > \frac{1}{2}x - 2$	$13 \leq \frac{10 - z}{4}$

How many inequalities can you find whose solution is shown on this number line?



Include examples with a on both sides of the inequality sign, integer and fractional coefficients.

Solve equations with algebraic fractions H

Notes and guidance

This step starts by exploring equations and inequalities that reduce to linear form. Students will need to have completed the higher steps on quadratics in the Autumn term algebra block to access the later questions in this step. Only quadratics that factorise will be covered at this stage. Students will revisit factorisation and quadratic equations in Year 11 “Expanding and Factorising” and you may leave this step until then if appropriate.

Key vocabulary

Equation	Solve	Solution
Verify	Quadratic	Factorise

Key questions

How can you make the equation simpler?
When multiplying an equation by an integer, why do you need to multiply every term?
How do you factorise a quadratic expression?
Why do you need to equate a quadratic expression to zero to solve a quadratic equation?

Exemplar Questions

What’s the same and what’s different about solving these equations?

- $\frac{(2a+8)}{5} = a + 1$
- $\frac{2a+8}{a+1} = 5$

Eva is solving the equation $\frac{1}{3m} + \frac{1}{2m} = \frac{5}{36}$

She writes $\frac{1}{3m} + \frac{1}{2m} = \frac{1}{5m}$ so $\frac{1}{5m} = \frac{5}{36}, 25m = 36, m = \frac{36}{25}$

Explain Eva’s mistake and find the correct value of m.

Solve the equations.

- $\frac{a}{5} + \frac{2a}{7} = \frac{34}{35}$
- $\frac{1}{n} - \frac{1}{5n} = \frac{1}{5}$
- $\frac{4}{a} - \frac{3}{2a} = \frac{1}{2}$
- $\frac{x+2}{5} + \frac{x+4}{3} = 6$
- $\frac{x-1}{4} + \frac{x-6}{2} = 2$
- $\frac{x+3}{2} - \frac{x-2}{4} = 5$

By simplifying to a quadratic, show that the equation has two integer solutions.

$$\frac{4}{x+6} + \frac{3}{x+1} = 1$$

Check your solutions by substituting into the given equation.

Given that $2x + 1 : 13 - 4x = 4 : 7$, find the value of x.

Represent numbers algebraically

Notes and guidance

Students have met the idea of representing numbers in general form in Year 9, but this complex idea may need reintroducing here. The key learning is that ak is a multiple of integer k when k is an integer. In particular, in the special case when $a = 2$ then $2k$ represents an even number and $2k \pm 1$ represents an odd number. Understanding this can be supported by the use of manipulatives and/or bar models.

Key vocabulary

General	Integer	Factor
Mutliple	Odd	Even

Key questions

What are the properties of even numbers?
What can you say about the product of an even number and any other number?
Why must e.g. $10k$ be even? What else can you deduce about $10k$?
Why might e.g. $3k$ be even or not?

Exemplar Questions

m is odd. What, if anything, can you say about the numbers represented by these expressions?

$m + 1$	$2m$	$3m$	$4m + 1$	$m + 4$

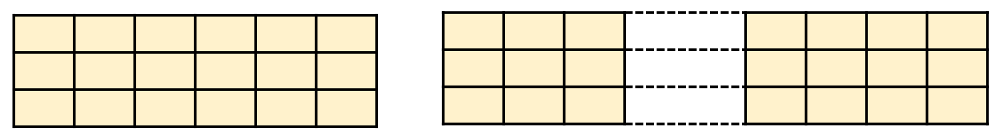
k is even. What, if anything, can you say about the numbers represented by these expressions?

$k + 1$	$2k$	$3k$	$4k + 1$	$k + 4$

n is an integer. What, if anything, can you say about the numbers represented by these expressions?

$n + 1$	$2n$	$n - 1$	$2n + 1$	$2n - 1$

Explain why both representations illustrate multiples of 3



How can you use these to show that the sum of two multiples of 3 is also a multiple of 3?
How can you show this result algebraically?

Are the statements always, sometimes or never true?

- The sum of two multiples of 4 is a multiple of 8
- The sum of a multiple of 4 and a multiple of 6 is even.
- The difference between two even numbers is odd.

Show your results using algebra.

Algebraic proof

Notes and guidance

Although formal proof is not required at Foundation tier, much of this step should be accessible to students aiming for a grade 5 as the idea of proof is introduced gradually through simple developing of arguments and/or the use of counterexamples. The focus remains on generalising numbers in algebraic form. More complex proof is dealt with in the Year 11 block ‘Show that’.

Key vocabulary

Prove	Show	Justify
Example	Counterexample	

Key questions

What’s the difference between a demonstration and a proof?
How does looking for factors of expressions help us to decide what they might be multiples of?
Why is one counterexample enough to disprove a statement but one example not enough to prove a statement?

Exemplar Questions

k is a positive integer.
Are these statements always, sometimes or never true?

$6k$ is a multiple of 3	$k + 2$ is even	$4k + 1$ is odd
$3k$ is a factor of $5k$	$k^2 > k$	$(k + 1)^2 - k^2$is odd

Use examples, counterexamples and algebraic methods to prove your answers.

Here are two students’ attempts at proving that the difference between two odd numbers is even. Find the mistakes and write a correct proof.

Odd numbers can be written in the form $2m + 1$ $2m + 1 \ - (2m + 1) = 0$ 0 is even So the difference between two odds in even.	Let the odd numbers be $2m + 1$ and $2n + 1$ $2m + 1 \ - 2n + 1 = 2m - 2n + 2$ $= 2(m - n + 1)$ Which is a multiple of 2, so the difference between two odds in even.

Spot the mistake in this “proof” that 1 = 2

Let $x = y$
So $x - y = 0$
Multiplying both sides by 2, $2x - 2y = 0$
As both expressions are equal to $0, x - y = 2(x - y)$
Rewriting this, $1(x - y) = 2(x - y)$
Dividing both sides by $x - y$ gives $1 = 2$