Indices and Roots

Year 10

#MathsEveryoneCan

WRM – Year 10 Scheme of Learning

	Week 1	Week 2	Week 3	Week 4	Week 5	Week 6	Week 7	Week 8	Week 9	Week 10	Week 11	Week 12
Autumn	**Similarity**						**Developing Algebra**					
	Congruence, similarity and enlargement			Trigonometry			Representing solutions of equations and inequalities			Simultaneous equations		
Spring	**Geometry**						**Proportions and Proportional Change**					
	Angles & bearings		Working with circles		Vectors		Ratios & fractions		Percentages and Interest		Probability	
Summer	**Delving into data**				**Using number**						**Expressions**	
	Collecting, representing and interpreting data				Non-calculator methods		Types of number and sequences		Indices and Roots		Manipulating expressions	

Summer 2: Using Number/Expressions

Weeks 1 and 2: Types of Number and Sequences

This block again mainly revises KS3 content, reviewing prime factorisation and associated number content such as HCF and LCM. Sequences is extended for Higher Tier to include surds and finding the formula for a quadratic sequence.

National curriculum content covered:

- consolidating subject content from key stage 3:
  - factors, multiples, primes, HCF and LCM
  - describe and continue sequences
- recognise and use sequences of triangular, simple arithmetic progressions, Fibonacci type sequences, quadratic sequences, and simple geometric progressions (r^n where n is an integer, and r is a positive rational number **{or a surd}) {and other sequences}**
- deduce expressions to calculate the nth term of linear **{and quadratic}** sequence

Weeks 3 and 4: Indices and roots

This block consolidates the previous two blocks focusing on understanding powers generally, and in particular in standard form. Negative and fractional indices are explored in detail. Again, much of this content will be familiar from KS3, particularly for Higher tier students, so this consolidation material may be covered in less than two weeks allowing more time for general non-calculator and problem-solving practice. To consolidate the index laws, these can be revisited in the next block when simplifying algebraic expressions.

National curriculum content covered:

- recognise and use sequences of square and cube numbers
- **{estimate powers and roots of any given positive number}**
- calculate with roots, and with integer **{and fractional}** indices
- calculate with numbers in standard form $A \times 10^n$, where $1 \leq A < 10$ and n is an integer
- simplifying expressions involving sums, products and powers, including the laws of indices

Weeks 5 and 6: Manipulating expressions

This final block of year 10 builds on the Autumn term learning of equations and inequalities, providing revision and reinforcement for Foundation tier students and an introduction to algebraic fractions for those following the Higher tier. This also allows all students to revise fraction arithmetic to keep their skills sharp. Algebraic argument and proof are considered, starting with identities and moving on to consider generalised number.

National curriculum content covered:

- simplify and manipulate algebraic expressions (including those involving surds **{and algebraic fractions}** by factorising quadratic expressions of the form $x^2 + bx + c$
- know the difference between an equation and an identity
- argue mathematically to show algebraic expressions are equivalent, and use algebra to support and construct arguments **{and proofs}**

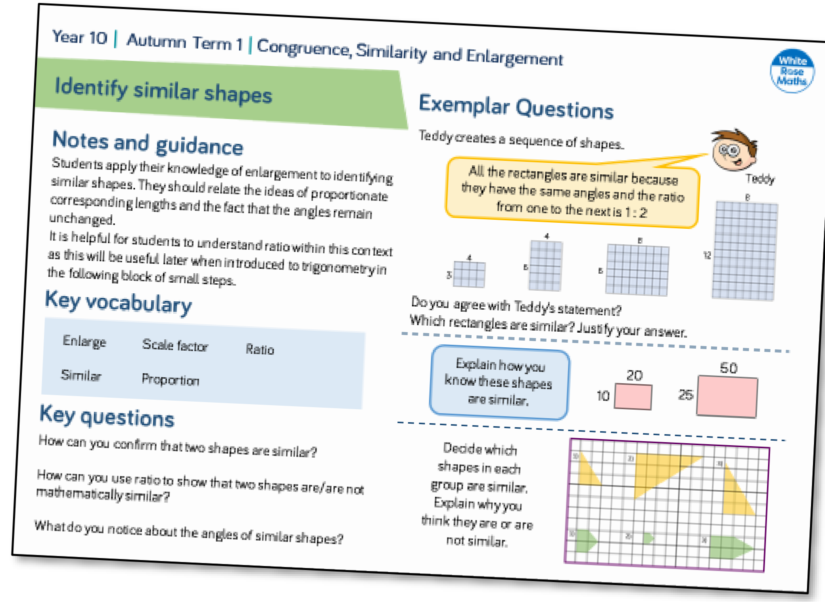
Why Small Steps?

We know that breaking the curriculum down into small manageable steps should help students to understand concepts better. Too often, we have noticed that teachers will try and cover too many concepts at once and this can lead to cognitive overload. We believe it is better to follow a "small steps" approach.

As a result, for each block of content in the scheme of learning we will provide a "small step" breakdown. ***It is not the intention that each small step should last a lesson – some will be a short step within a lesson, some will take longer than a lesson.*** We would encourage teachers to spend the appropriate amount of time on each step for their group, and to teach some of the steps alongside each other if necessary.

What We Provide

- Some ***brief guidance*** notes to help identify key teaching and learning points
- A list of ***key vocabulary*** that we would expect teachers to draw to students' attention when teaching the small step,
- A series of ***key questions*** to incorporate in lessons to aid mathematical thinking.
- A set of questions to help ***exemplify*** the small step concept that needs to be focussed on.



Year 10 | Autumn Term 1 | Congruence, Similarity and Enlargement

Identify similar shapes

Notes and guidance

Students apply their knowledge of enlargement to identifying similar shapes. They should relate the ideas of proportionate corresponding lengths and the fact that the angles remain unchanged.

It is helpful for students to understand ratio within this context as this will be useful later when introduced to trigonometry in the following block of small steps.

Key vocabulary

Enlarge	Scale factor	Ratio
Similar	Proportion	

Key questions

How can you confirm that two shapes are similar?

How can you use ratio to show that two shapes are/are not mathematically similar?

What do you notice about the angles of similar shapes?

Exemplar Questions

Teddy creates a sequence of shapes.

All the rectangles are similar because they have the same angles and the ratio from one to the next is 1 : 2

Teddy

Do you agree with Teddy's statement?
Which rectangles are similar? Justify your answer.

Explain how you know these shapes are similar.

Decide which shapes in each group are similar. Explain why you think they are or are not similar.

- These include reasoning and problem-solving questions that are fully integrated into the scheme of learning. Depending on the attainment of your students, you many wish to use some or all of these exemplars, which are in approximate order of difficulty. Particularly challenging questions are indicated with the symbol .
- For each block, we also provide ideas for key representations that will be useful for all students.

Some of the small steps are in **bold** and labelled with H to indicate this is Higher tier GCSE content. We would encourage teachers to use these with as many students as possible – if you feel your class can access any particular small step, then please include it in your planning. Steps that review content covered at Key Stage 3 are labelled R .

Indices and Roots

Small Steps

- Square and Cube numbers (R)
- Calculate higher powers and roots
- Powers of ten and standard form (R)
- The addition and subtraction rules for indices (R)
- Understand and use the power zero and negative indices
- Work with powers of powers
- **Understand and use fractional indices** (H)
- Calculate with numbers in standard form (R)

(H) denotes Higher Tier GCSE content

(R) denotes 'review step' – content should have been covered at KS3

Square and Cube numbers

R

Notes and guidance

Students are usually less familiar with the cube numbers than the square numbers, so regular revisiting is important. Linking to area and volume is helpful, as is showing how the cubes can be found easily from the squares. It is helpful if students can commit the first 12 square numbers to memory and at least the first five cubes, particularly to support the finding of square and cube roots. Revisiting Pythagoras' theorem may also be useful.

Key vocabulary

Square, Cube, Root, Prime, Prime factorisation, Integer

Key questions

What's the difference between the square of a number and the square root of a number?

How do we find squares/cubes/roots on a calculator?

What do we know about a number if its square root is an integer?

Exemplar Questions

Continue the sequences as far as you can with the square and cube numbers you know.

n	1	2	3	- - - - -
n^2	1	4	9	- - - - -
n^3	1	8	27	- - - - -

How can you work out the next terms in the sequences

- using a calculator?
- using a written method?

Which of the numbers have integer square roots, integer cube roots or both?

81 | 64 | 125 | 1 | 1000 | 196

Dora says you can tell if a number is square by looking at its prime factorisation.

$324 = 2 \times 2 \times 3 \times 3 \times 3 \times 3$
$= (2 \times 3 \times 2)^2$
$= 18^2$

Use prime factorisation to determine if these numbers are square.

- 256
- 216
- 625
- 400
- 1024
- 729

If they are square, state their square roots.

Are any of the numbers cube numbers?

Higher powers and roots

Notes and guidance

The key point of this step is to ensure students are familiar with the notation rather than performing all the calculations by hand. It is often far more appropriate to use a calculator to work out these values and students may need to be taught how to use the x^n (or equivalent) key. The Higher tier requires students to estimate higher powers and roots, so this has been included as an extension question here. Familiarity with higher powers will help with fractional indices later.

Key vocabulary

Root	Power	Index/Indices
Fourth root	Estimate	Exponent

Key questions

What does "to the power (e.g.) 4" mean? Can you say this another way?

How do you use a calculator to quickly find a number to the 4th, 5th… power?

How do you use a calculator to find the 4th, 5th… root of a number?

Exemplar Questions

Match the cards of equal value.

8^2 | 2^6 | 4^4 | 16^2 | 2^8 | 4^3 | 64^1

Using a calculator, or otherwise, work out the values.

- 4^5
- 5^4
- 2^{10}
- $(-2)^6$
- 0.1^3
- $\sqrt[4]{6\,561}$

Using a calculator, or otherwise, work out the values.

- $(-1)^1$
- $(-1)^2$
- $(-1)^3$
- $(-1)^4$
- $(-1)^5$

What do you notice?

Write down the value of

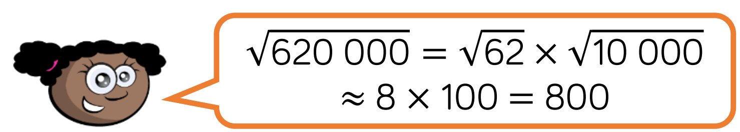
- $(-1)^6$
- $(-1)^{11}$
- $(-1)^{204}$

$2^1 = 2$
$2^2 = 4$
$2^3 = 8$
$2^4 = 16$
$2^5 = 32$

The last digit of powers of 2 follow a repeating pattern: 2, 4, 8, 6, 2, 4 …

Investigate the pattern formed by last digits of

- powers of 5
- powers of 10
- powers of 3
- powers of 8



Use Whitney's strategy, or another method, to estimate:

- $\sqrt{890}$
- $\sqrt{39\,000}$
- 611^3
- $\sqrt[3]{28\,000\,000}$
- 18.5^4

Standard Form

R

Notes and guidance

As students have been working with numbers in standard form since Year 8, the conversions chosen for the exemplar questions are in the context of simple calculations rather than just "convert" questions. It is always useful to look at numbers in context e.g. populations, land areas, atoms etc. to provide meaning. This is also a good chance to revisit words like million, billion etc. Some students may need to be supported with a place value chart.

Key vocabulary

Standard form	Power	Index/Indices
Exponent	Million/Billion	

Key questions

How can you tell if a number is written in standard form or not?

How can you convert a number greater than 1 /less than 1 to/from standard form?

What numbers do you look at first when comparing numbers written in standard form?

Exemplar Questions

Which of these numbers are not in standard form? Explain why.

3.7×10^{-2}	$8.5 \times 10^{0.5}$	0.5×10^{8}	11×10^{4}

Work out the calculations, giving your answers in standard form.

- $3.7 \times 1\,000$
- 26×100
- $10 \times 1.4 \times 1\,000$
- 0.3×100
- $0.16 \times 10\,000$
- $35.7 \times 100\,000$
- $1.5 \div 10$
- $6.8 \div 1\,000$
- $1.407 \div 100$
- $93 \div 1\,000$
- $204 \div 10\,000$
- $0.55 \div 1\,000$

The mass of Saturn is 5.6×10^{26} kg.
The mass of Uranus is 8.7×10^{25} kg.
Which planet is heavier? Explain how you know.

Work out the calculations, giving your answers in standard form.

- $3\,000 \times 200$
- $60\,000 \div 20$
- $5\,000 \times 4\,000$
- $400 \div 2\,000$
- $400 \div 20\,000$
- $40\,000^2$
- 70×0.0001
- $20 \div 0.001$
- 0.3^3

Identify the larger number in each pair.

$30\,000$	3×10^{5}
0.006	5×10^{-3}
1 billion	8×10^{8}
6.1×10^{-2}	0.016
0.0001	1×10^{-5}
4×10^{17}	7×10^{14}

Addition/Subtraction indices R

Notes and guidance

Students have met the rules of indices at key stage 3, so this review step is designed to reinforce their prior learning. It is helpful to look at questions with both numerical and algebraic bases, and also to include questions that involve both the addition and subtraction of indices. Negative results could be included here if appropriate, but these are covered in detail in the next step. It is always worth reminding students that a and a^1 are equivalent.

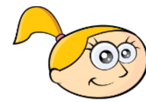
Key vocabulary

Base	Index/Indices	Simplify
Power	Exponent	

Key questions

What is the difference between a base and an index?
How can you simplify the multiplication of two terms involving indices if they have the same base?
Can you use the same rule if the bases are different?
Why is (e.g.) $a^6 \div a = a^5$ when there is no index on the second term?

Exemplar Questions



By writing out the calculation in full, show that Eva is wrong.

Write these expressions as a single power of 3

- $3^5 \times 3^2$
- $3^5 \times 3^{12}$
- $3^5 \times 3$
- $3^5 \times 3^5 \times 3^5$

$$2^6 \div 2^3 = \frac{2^6}{2^3} = \frac{2 \times 2 \times 2 \times 2 \times 2 \times 2}{\square} = 2^{\square}$$

Complete the calculation.
Which is the correct generalisation?

$a^m \div a^n = a^{m \div n}$ or $a^m \div a^n = a^{m-n}$

Write these expressions as single powers of a.

- $a^{10} \div a^2$
- $a^6 \div a$
- $a^7 \div a^5 \times a^2$
- $a^7 \div (a^5 \times a^2)$

Find the values of the letters.

- $2^5 \times 2^a = 2^{10}$
- $2^8 \div 2^b = 2^5$
- $2^5 \times 2^c \div 2^3 = 2^2$
- $2^d \times 2^d = 2^8$
- $2^9 \div 2^e = 2^8$
- $2^3 \times 2 \times 2^f = 2^8$

Do you agree?
Why or why not?

Zero and negative indices

Notes and guidance

The common misconception that a number raised to the power zero gives the result zero needs to be addressed and revisited often. Similarly, students often confuse negative indices with negative numbers, so deriving the rules to provide meaning is a helpful strategy, as is comparing with earlier experience of standard form. Using a calculator to verify results is useful here, and it is useful to link to the previous step, particular using $a^m \div a^n = a^{m-n}$ where $m - n \leq 0$

Key vocabulary

Base	Index/Indices	Negative
Power	Exponent	Simplify

Key questions

What is the result when you divide a number by itself?
What is the value of a^0 for any value of a?
Can you use negative numbers when finding powers? Is this the same as, or different from negative indices?
How do negative powers of 10 connect with standard form?

Exemplar Questions

$5^3 \div 5^3 = 1$ as any number divided by itself is 1

But $5^3 \div 5^3 = 5^{3-3} = 5^0$ So $5^0 = 1$

Use Rosie and Mo's reasoning to write down the answers to

- 6^0
- 7^0
- $\left(\frac{1}{2}\right)^0$
- 37^0
- $2^3 \times 2^0$

I know from standard form that $10^{-1} = \frac{1}{10}$ and $10^{-2} = \frac{1}{10^2}$, so in the same way $3^{-1} = \frac{1}{3}$

Amir is correct. Use his reasoning to match the cards of equal value.

3^{-2}	3×-2	3^0	3^2	3^{-3}	0^3	-3^3
-6	0	$\frac{1}{9}$	-27	1	$\frac{1}{27}$	9

All the statements are wrong. Correct them.

$5^0 = 0$	$3 \times 2^{-1} = \frac{1}{6}$	$4^{-1} = -4$	$2^3 \times 2^{-2} = 2^{-6}$
$4 \times 8^{-1} = \frac{1}{32}$	$8^9 \div 8^3 = 1^6$	$6^3 \times 6 = 6$	$6^{-2} = \frac{1}{12}$

Powers of powers

Notes and guidance

Some students will have met $(a^b)^c = a^{bc}$ at KS3, but this may well be new to Foundation tier students. Again, deriving the law from writing calculations in full helps understanding and retention. Students should also use this law in conjunction with those from the previous step, deciding which rule to use in which situation. This is a good point at which to consider questions of the form $(3x^5)^4$ as well as e.g. $5x^3y^2 \times 3x^4y^5$ and divisions.

Key vocabulary

Base	Index/Indices	Negative
Power	Exponent	Simplify

Key questions

Will $(a^b)^c$ be the same as, or different from $(a^c)^b$? Why?

Why do we need to be careful with expressions like $(6x^2)^3$?

How would you start solving an index question that involves more than one operation?

Exemplar Questions

$(2^5)^3 = 2^5 \times 2^5 \times 2^5 = 2^{5+5+5} = 2^{15}$

Use the same reasoning to work out

- $(4^3)^2$
- $(6^2)^4$
- $(9^4)^3$

What is the "quick way" of finding the answers to the questions?

Generalise $(a^b)^c = a^{\square}$

Write down the answers to

- $(5^6)^4$
- $(6^8)^5$
- $(7^6)^a$
- $(b^9)^3$
- $(2^{-2})^5$

Find the values of the letters.

$(3^{10})^4 = 3^a$	$(6^a)^4 = 6^{16}$	$(4^5)^3 \times 4^b = 4^{27}$
$(5^c)^2 \times 5^4 = 5^{10}$	$7^d \div (7^3)^4 = 1$	$(7^{-2})^e = 7^8$

The answer to a question is a^{20}.

Find some different possible questions, using as many indices rules as you can.

Which is the correct answer to $(2c^5)^3$?

- $6c^{15}$
- $8c^{35}$
- $8c^{15}$
- $6c^{53}$

Write these expressions as single powers of 2

- 4
- 4^3
- 8^5
- 8^{-1}
- 16^{-2}
- $4^2 \times 8^{-2}$
- 3^0

Fractional Indices

H

Notes and guidance

As well as covering the meaning of indices that are unit fractions, this step extends understanding to look at non-unit fractions though 'reversing' the powers of powers law. Familiarity with square, cube and higher roots is vital here. As students have already met irrational numbers, this is a good point at which to revisit surds e.g. by linking $2^{\frac{1}{2}}$ and $\sqrt{2}$ or $8^{\frac{1}{2}} = \sqrt{8} = 2\sqrt{2}$ etc.

Key vocabulary

Power	Index	Root
Unit-fraction	Non-unit fraction	

Key questions

What's the difference between "finding one half" and "raising to the power one half"?

If you know the value of (e.g.) $x^{\frac{1}{3}}$, how can you find the value of $x^{-\frac{1}{3}}$?

What are the steps in finding e.g. $125^{-\frac{2}{3}}$?

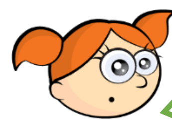
Exemplar Questions

$9^{\frac{1}{2}} \times 9^{\frac{1}{2}} = 9^1$

So $9^{\frac{1}{2}} = \sqrt{9} = 3$

Use Annie's reasoning to work out

- $81^{\frac{1}{2}}$
- $64^{\frac{1}{2}}$
- $64^{\frac{1}{3}}$
- $64^{-\frac{1}{3}}$
- $16^{\frac{1}{4}}$



$16^{\frac{3}{4}} = \left(16^{\frac{1}{4}}\right)^3$

$= \left(\sqrt[4]{16}\right)^3$

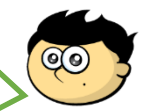
$= 2^3 = 8$

$16^{\frac{3}{4}} = (16^3)^{\frac{1}{4}}$

$= \sqrt[4]{16^3}$

$= \sqrt[4]{4\,096}$

$= 8$



Which method do you prefer?

Showing all your working, find the values of

- $81^{\frac{3}{2}}$
- $64^{\frac{2}{3}}$
- $100^{-\frac{5}{2}}$
- $125^{-\frac{2}{3}}$
- $32^{0.8}$
- $16^{\frac{5}{4}}$
- $\left(\frac{64}{125}\right)^{\frac{2}{3}}$
- $\left(\frac{64}{125}\right)^{-\frac{2}{3}}$
- $\left(\frac{81}{16}\right)^{\frac{3}{4}}$
- $\left(\frac{16}{25}\right)^{-1.5}$

Find the values of the letters.

$16^{\frac{3}{4}} \times 64^{\frac{2}{3}} = 2^a$

$100^{\frac{3}{2}} \times 1\,000^{\frac{2}{3}} = 10^b$

$72^{\frac{1}{2}} = c\sqrt{d}$

$9^{-\frac{1}{2}} \times 27^{\frac{2}{3}} = 3^{e+2}$

$8^{\frac{3}{2}} = f\sqrt{2}$

Standard Form Calculations R

Notes and guidance

This step revises KS3 work, and includes solving problems both with and without a calculator. Non-calculator work in particular is useful in reinforcing the laws of indices from earlier in this block. Care needs to be taken to establish the different ways of working when adding and subtracting rather than multiplying or dividing. It may be useful to remind students how to round to significant figures and how this works with numbers given in standard form.

Key vocabulary

Standard form	Power	Index/Indices
Exponent	SCI/EXP	Scientific Notation

Key questions

How do you input a number in standard form in your calculator? Is it the same or different if the power of 10 is negative?

What's the same and what's different about adding/subtracting and multiplying/dividing standard form numbers without a calculator?

Exemplar Questions

Correct the errors in these calculations.
Give all answers in standard form.

- $(3 \times 10^5) \times 4 = 12 \times 10^{20}$
- $(8 \times 10^{10}) \div 2 = 4 \times 10^5$
- $(7 \times 10^4) \times (6 \times 10^3) = 42 \times 10^7$
- $(3 \times 10^4) + (2 \times 10^5) = 5 \times 10^9$
- $(8.4 \times 10^6) \div (2 \times 10^3) = 4.2 \times 10^2$
- $(6 \times 4) \times 10^0 = 0$

Solve the problems without using a calculator, giving all answers in standard form unless otherwise stated.

- What number is one million times bigger than 9×10^7?
- What number is one million times smaller than 9×10^7?
- What number is one million greater than 9×10^7?
- What number is one million smaller than 9×10^7?
- Show that $4 \times 10^{-2} + 5 \times 10^{-3} = \frac{9}{200}$
- Work out $(3.16 \times 10^4) \times (4 \times 10^{-2})$
- Work out $(3.16 \times 10^4) \div (4 \times 10^{-2})$

The mass of the Sun is 1.989×10^{30} kg.
The mass of the Earth is 5.792×10^{24} kg.
How many times heavier than the Earth is the Sun?
Give your answer in standard form, correct to 3 significant figures.