Types of Number and Sequences

Year 10

#MathsEveryoneCan

White Rose Maths

WRM – Year 10 Scheme of Learning

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	Week 1	Week 2	Week 3	Week 4	Week 5	Week 6	Week 7	Week 8	Week 9	Week 10	Week 11	Week 12
Autumn	**Similarity**						**Developing Algebra**					
	Congruence, similarity and enlargement			Trigonometry			Representing solutions of equations and inequalities			Simultaneous equations		
Spring	**Geometry**						**Proportions and Proportional Change**					
	Angles & bearings		Working with circles		Vectors		Ratios & fractions		Percentages and Interest		Probability	
Summer	**Delving into data**				**Using number**						**Expressions**	
	Collecting, representing and interpreting data				Non-calculator methods		Types of number and sequences		Indices and Roots		Manipulating expressions	

Summer 2: Using Number/Expressions

Weeks 1 and 2: Types of Number and Sequences

This block again mainly revises KS3 content, reviewing prime factorisation and associated number content such as HCF and LCM. Sequences is extended for Higher Tier to include surds and finding the formula for a quadratic sequence.

National curriculum content covered:
- consolidating subject content from key stage 3:
  - factors, multiples, primes, HCF and LCM
  - describe and continue sequences
- recognise and use sequences of triangular, simple arithmetic progressions, Fibonacci type sequences, quadratic sequences, and simple geometric progressions ($r^n$where n is an integer, and r is a positive rational number **{or a surd})** **{and other sequences}**
- deduce expressions to calculate the nth term of linear **{and quadratic}** sequence

Weeks 3 and 4: Indices and roots

This block consolidates the previous two blocks focusing on understanding powers generally, and in particular in standard form. Negative and fractional indices are explored in detail. Again, much of this content will be familiar from KS3, particularly for Higher tier students, so this consolidation material may be covered in less than two weeks allowing more time for general non-calculator and problem-solving practice. To consolidate the index laws, these can be revisited in the next block when simplifying algebraic expressions.

National curriculum content covered:
- recognise and use sequences of square and cube numbers
- **{estimate powers and roots of any given positive number}**
- calculate with roots, and with integer **{and fractional}** indices
- calculate with numbers in standard form $A \times 10^n$, where $1 \leq A < 10$ and n is an integer
- simplifying expressions involving sums, products and powers, including the laws of indices

Weeks 5 and 6: Manipulating expressions

This final block of year 10 builds on the Autumn term learning of equations and inequalities, providing revision and reinforcement for Foundation tier students and an introduction to algebraic fractions for those following the Higher tier. This also allows all students to revise fraction arithmetic to keep their skills sharp. Algebraic argument and proof are considered, starting with identities and moving on to consider generalised number.

National curriculum content covered:
- simplify and manipulate algebraic expressions (including those involving surds **{and algebraic fractions}** by factorising quadratic expressions of the form $x^2 + bx + c$
- know the difference between an equation and an identity
- argue mathematically to show algebraic expressions are equivalent, and use algebra to support and construct arguments **{and proofs}**

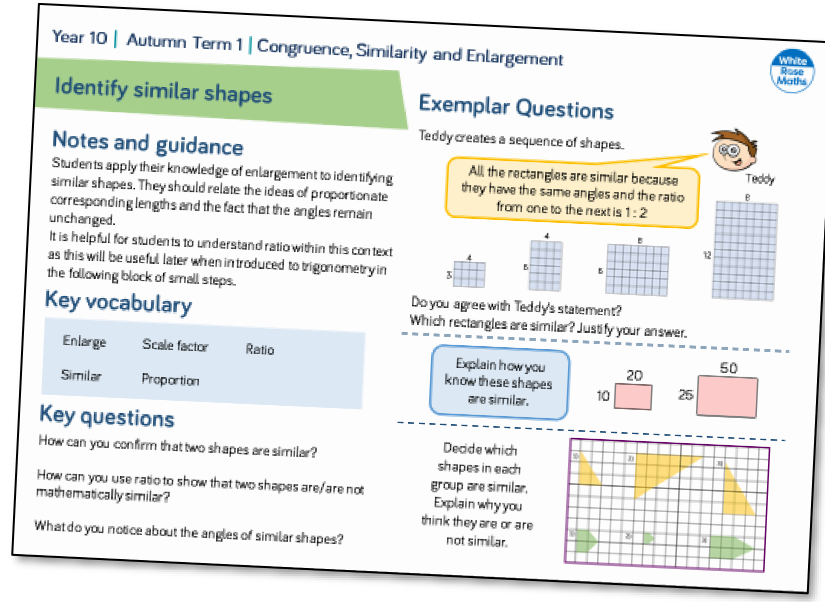
Why Small Steps?

We know that breaking the curriculum down into small manageable steps should help students to understand concepts better. Too often, we have noticed that teachers will try and cover too many concepts at once and this can lead to cognitive overload. We believe it is better to follow a "small steps" approach.

As a result, for each block of content in the scheme of learning we will provide a "small step" breakdown. ***It is not the intention that each small step should last a lesson – some will be a short step within a lesson, some will take longer than a lesson.*** We would encourage teachers to spend the appropriate amount of time on each step for their group, and to teach some of the steps alongside each other if necessary.

What We Provide

- Some ***brief guidance*** notes to help identify key teaching and learning points
- A list of ***key vocabulary*** that we would expect teachers to draw to students' attention when teaching the small step,
- A series of ***key questions*** to incorporate in lessons to aid mathematical thinking.
- A set of questions to help ***exemplify*** the small step concept that needs to be focussed on.


Year 10 | Autumn Term 1 | Congruence, Similarity and Enlargement

Identify similar shapes

Notes and guidance

Students apply their knowledge of enlargement to identifying similar shapes. They should relate the ideas of proportionate corresponding lengths and the fact that the angles remain unchanged.

It is helpful for students to understand ratio within this context as this will be useful later when introduced to trigonometry in the following block of small steps.

Key vocabulary

Enlarge Scale factor Ratio

Similar Proportion

Key questions

How can you confirm that two shapes are similar?

How can you use ratio to show that two shapes are/are not mathematically similar?

What do you notice about the angles of similar shapes?

Exemplar Questions

Teddy creates a sequence of shapes.

All the rectangles are similar because they have the same angles and the ratio from one to the next is 1 : 2

Teddy

Do you agree with Teddy's statement? Which rectangles are similar? Justify your answer.

Explain how you know these shapes are similar.

Decide which shapes in each group are similar. Explain why you think they are or are not similar.

- These include reasoning and problem-solving questions that are fully integrated into the scheme of learning. Depending on the attainment of your students, you many wish to use some or all of these exemplars, which are in approximate order of difficulty. Particularly challenging questions are indicated with the symbol .
- For each block, we also provide ideas for key representations that will be useful for all students.

Some of the small steps are in **bold** and labelled with H to indicate this is Higher tier GCSE content. We would encourage teachers to use these with as many students as possible – if you feel your class can access any particular small step, then please include it in your planning. Steps that review content covered at Key Stage 3 are labelled R .

Types of Number and Sequences

Small Steps

- Understand the difference between factors and multiples (R)
- Understand primes and express a number as a product of its prime factors (R)
- Find the HCF and LCM of a set of numbers (R)
- Describe and continue arithmetic and geometric sequences
- Explore other sequences
- **Describe and continue sequences involving surds** (H)
- Find the rule for the n^{th} term of a linear sequence (R)
- **Find the rule for the n^{th} term of a quadratic sequence** (H)

H	denotes Higher Tier GCSE content
R	denotes 'review step' – content should have been covered at KS3

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Factors and multiples

R

Notes and guidance

The main emphasis of this step is to review the difference between a factor and a multiple. Building from previous years, students will explore both factors of numbers and algebraic expressions. The area model is useful in considering factors and links well to factors of algebraic terms. Ensure that students are also exposed to non-examples and non-standard examples of factors and multiples e.g. 0.5 is not a factor of 1

Key vocabulary

Integer	Factor	Multiple
Area	Factorise	Prime

Key questions

Explain why a factor is different from a multiple.

Can negative numbers be multiples/factors?

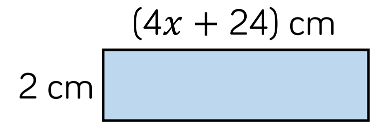
Can algebraic expressions be multiples/factors?

Can a fraction of a number be a multiple/factor?

Exemplar Questions

Using integers for length and width, find as many different rectangles with have an area of 48 cm^2. What are the factors of 48?

Eva draws a rectangle with an area of $8x + 48$ cm^2



Draw 3 other rectangles that have the same area as Eva's.
Eva says that $x + 6$ is a factor of $8x + 48$
She is correct. Explain why.

Decide whether the statements are true or false.
Justify your decisions.

- 49 is a multiple of 7
- y^2 is a multiple of y
- y is a factor of y^2
- $8x + 48x$ is a multiple of $8x$
- $8x + 48$ is a multiple of 8
- $8x + 48$ is a multiple of $8x$

Is this statement, always, sometimes or never true? Explain why.

If a is a factor of b, then b is a multiple of a

---

500 g pack of lemons cost £1.20
1 kg pack of sugar cost £0.80

Jack buys packs of lemon and sugar for his crepe store.
He spends exactly £20
How many kg of lemons and how many kg of sugar did he buy?

Product of prime factors

R

Notes and guidance

Students should already be familiar with expressing a number as a product of its prime factors, but some of the language (e.g. express, product) requires emphasis. It's worth ensuring that students understand the concept behind the procedure and make links between the original number and the product. Students should use their reasoning skills to make connections between the prime factor decomposition of related numbers.

Key vocabulary

Factor	Prime Factor	Factorise
Product	Express	Index form

Key questions

Explain what the following terms mean:
Factor of, Express, Product, Prime Factor, Factorise

Is there more than one way to factorise 300?

If there more than one way to express 300 as a product of prime factors? Does the order of the factors matter?

Exemplar Questions

Aisha has expressed a number as a product of its prime factors.

What is the number? $2^2 \times 3 \times 5^2$

$$36 = 2^2 \times 9$$

Dexter attempts to express 36 as a product of its prime factors.
What mistake has he made?

He now writes 36×24 as a product of prime factors.
Dexter considers the following two approaches:

$36 \times 24 = 864$ I can draw a prime factor tree for 864	I just need to draw a prime factor tree for 24

Which approach is the most efficient? Why?
Write 36×24 as a product of its prime factors.

Mo has written 400 as a product of its prime factors.

$$400 = 2^4 \times 5^2$$

Use this to write down the numbers as products of their prime factors:

- 200
- 50
- 100
- 80
- 16
- 800
- 4 000

$$375 = 5^x \times y$$

Show that $x \div y = 1$

HCF and LCM

R

Notes and guidance

Students may need reminders with this familiar topic as they can confuse the HCF with the LCM. They need to be careful to use prime factors when completing Venn diagrams, rather than just factors. Duplicating the common factors when filling in the intersection of the Venn diagram is a common error that may also need highlighting. Students may multiply all prime factors of both numbers when finding the LCM, so it is useful to demonstrate that this is often unnecessary.

Key vocabulary

Highest Common Factor/Lowest Common Multiple

Prime Factors   Product   Intersection

Key questions

What does common mean when looking at factors and multiples?

Why do we find the highest common factor but the lowest common multiple? Why isn't it the other way round?

What's the first step in completing a Venn diagram to find the HCF and LCM?

Exemplar Questions

Find the LCM of:

- 3 and 15
- 4 and 6
- 3 and 8
- 100 and 200
- 10 and 25
- 8 and 15

What do you notice about the LCM in each pair?

To find the LCM, multiply the numbers together

Is Ron correct? Explain your answer.

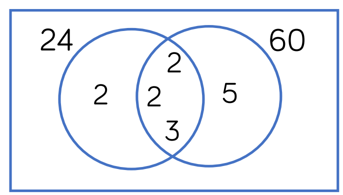
Rosie is making fried egg sandwiches.

Each sandwich contains one fried egg.

Eggs come in boxes of one dozen and bread rolls come in packs of 8

What is the least number of boxes of eggs and packs of bread rolls Rosie needs to buy so she has no ingredients left over?

Eva is using a Venn diagram to find the LCM and HCF of 24 and 60



$24 = \cancel{2} \times \cancel{2} \times 2 \times \cancel{3}$

$60 = \cancel{2} \times \cancel{2} \times \cancel{3} \times 5$

Explain why $2 \times 2 \times 3$ gives the HCF of 24 and 60

To find the LCM, Eva works out $2 \times 2 \times 2 \times 3 \times 2 \times 2 \times 3 \times 5$

What mistake has she made? Correct this and calculate the LCM.

Why does this method work?

Arithmetic/Geometric sequences

Notes and guidance

Students can have the misconception that a common ratio of a geometric sequence has to be a positive integer and so should work with examples of fractions, decimal and negatives. Revisiting compound interest is useful as students make links between this and geometric sequences. For high attainers the formula for the n^{th} term of a geometric sequence can be explored; finding the rule for arithmetic sequences is reviewed later in this block.

Key vocabulary

Arithmetic	Common difference	n^{th} term
Geometric	Common ratio	Term-to-term

Key questions

What is an arithmetic sequence? Why is a geometric sequence different?
What is a common ratio? Does this have to be a positive integer? Explain why not.
What's the difference between a term-to-term rule and a position-to-term rule?

Exemplar Questions

A: 3, 5, 7,… B: 3, 15, 75…

- What's the same and what's different about the two sequences?
- Write down the term-to-term rule for each sequence.
- Which one will have a term greater than 100 first?
- Write down the first four terms of two more geometric sequences, one of which is ascending and one descending. What's the common ratio for each sequence?

Continue each of the following sequences to generate one that is arithmetic and the other geometric.

- $\frac{1}{5}, \frac{1}{10}, \ldots$
- $-0.3, -3, \ldots$

Teddy saves £1 in January, £2 in February, £4 in March and continues to double his saving each month.
Is this model realistic for a whole year? Explain your answer.

State whether these statements are true or false. Justify your answer each time.

- It's impossible for a geometric sequence to alternate between positive and negative numbers.
- An arithmetic sequence has to be ascending or descending. It can't alternate between the two.
- If you start with a positive number, and the common ratio is 0.5, you will eventually reach 0

Explore other sequences

Notes and guidance

This small step provides an opportunity for students to explore less familiar sequences. They explore sequences such as those that oscillate, the triangular numbers and Fibonacci sequences. Square number and cube number sequences could be included but will be looked at again in the next block so could be omitted if time is short. Concrete manipulatives, such as multi-link cubes allow students to represent the sequences and support their reasoning.

Key vocabulary

Square	Triangular	Cube
Oscillate	Predict	Fibonacci

Key questions

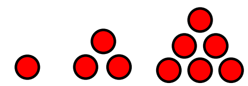
How can you represent the sequence using multi-link cubes? How does this help you justify your answer?

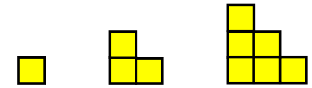
Is (e.g.) 5, 3, 5, 3, 5, 3…. a sequence? Explain the rule.

How do the square numbers differ from cube numbers?

Exemplar Questions

Here are two different ways of representing triangular numbers.





For each, draw pattern 4 What is the 4th triangular number?
What is the 10th triangular number? Explain how you found it.

Eva says: 20th triangular number is 20 + 19 + 18 + … + 3 + 2 + 1

Explain why Eva's calculation is correct.

Eva pairs the numbers in her calculation

$$(20 + 1) + (19 + 2) + (18 + 3)\ldots$$

She says that the 20th triangular number is the same as 21 × 10
Is Eva right? Explain your answer. Find the 100th triangular number.

1, 1, 2, 3, 5

4, 3, 7, 10, 17

Look at the two Fibonacci sequences. What's the same and what's different?

- Add together the first and last term. Compare with the middle term. What do you notice? Will this always happen?
- Write down 3 Fibonacci sequences which have 7 terms. Subtract the first term from the last term. Compare this with the middle term. What do you notice? Will this always happen?
- Explore further.

1, 1 + 3, 1 + 3 + 5, …

What's the 1 000th term in this sequence?
What's the 1 000 000th term?
Write your answers in standard form.

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Sequences involving surds

H

Notes and guidance

Students have already met simplifying and calculating with surds. They practise their skills in the context of both arithmetic and geometric sequences, either starting with surds, having a common difference/ratio that is a surd or both.

Key vocabulary

Simplest form	Surd	Common ratio
Common difference	Arithmetic	Geometric

Key questions

Does the method for finding the n^{th} term of a sequence change if it involves surds?

Why is simplification important when a sequence involves surds?

How can we work out the common ratio/common difference? How can we simplify this?

Exemplar Questions

The first term in a geometric sequence is 6

The common ratio of the sequence is $\sqrt{2}$

Write down the first five terms in the sequence in their simplest form.

If there are an even number of terms in the sequence, what proportion will be integers?

$$\sqrt{2},\ 2\sqrt{2},\ 3\sqrt{2},\ 4\sqrt{2}, \ldots$$

Is the sequence arithmetic or geometric? Find the rule for the n^{th} term.

Calculate the 20th number in the sequence, giving your answer in its simplest form.

$$\_\_\_,\ \sqrt{8}+1,\ 3\sqrt{2}+4, \ldots$$

Simplify the second term of this arithmetic sequence.

Find the common difference of the sequence.

Show that the first term of the sequence is $\sqrt{2}-2$

Categorise the sequences into arithmetic and geometric.

For each sequence find the common difference or the common ratio, and the next term.

- $3,\ 3+\sqrt{5},\ 3+2\sqrt{5},\ 3+3\sqrt{5},\ \_\_\_$
- $2,\ 2\sqrt{2},\ 4,\ 4\sqrt{2},\ 8,\ \_\_\_$
- $2\sqrt{3},\ 12,\ 24\sqrt{3},\ 144,\ \_\_\_$
- $5+\sqrt{7},\ 1-3\sqrt{7},\ -3-7\sqrt{7},\ -7-11\sqrt{7},\ \_\_\_$

n^{th} term of a linear sequence R

Notes and guidance

This small step reviews prior learning. Teachers might consider using sequences with decimal/fractional differences to extend this. The key here is that students understand the connection between the sequence and the associated multiplier. Use of descending sequences can also be used to prompt discussion about the multiplier.

Key vocabulary

Rule	Term-to-term	Position-to-term
Linear	Non-linear	Coefficient

Key questions

What does n represent?

How does the constant difference relate to the coefficient of n?

Why is the coefficient of n in a descending sequence negative?

Exemplar Questions

All of these sequences follow a rule in the form $5n +$ __ or $5n -$ __
Explain why this is the case.

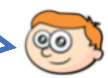
- 2, 7, 12, 17, …
- −20, −15, −10, −5, …
- 13.2, 18.2, 23.2, 28.2, …

Find the n^{th} term for each sequence.

$f,\ f+2,\ f+4,\ f+6, \ldots$

Eva 

n^{th} term = +2 each time

Ron 

n^{th} term $= f + 2n - 2$

Amir 

n^{th} term $= f + 2$

Dora 

n^{th} term $= f + 2n$

Students are finding the n^{th} term of the sequence.
Who's right? Explain why.

7, __, __, __, __, __, 31

Jack is finding the n^{th} term of this linear sequence. He starts by forming an equation for the 1^{st} term.

n^{th} term $= an + b$
1^{st} term $= 1a + b$
$7 = a + b$

- Write a second equation using the 7^{th} term.
- Solve the simultaneous equations to find a and b
- Use the rule for the n^{th} term to find the missing values in the sequence.
- Show that 217 is not in the sequence. Show that 75 is in the sequence.

n^{th} term quadratic sequence (H)

Notes and guidance

Students must be secure in finding the n^{th} term of a linear sequence before starting this small step. Through exploration, students should spot the link between the second difference and the coefficient of n^2. Once this is established, students can then compare an^2 with the given quadratic sequence (by finding the difference) allowing them to find $bn + c$. If appropriate, simultaneous equations and proof can be interleaved here.

Key vocabulary

Term	Difference	Linear
Quadratic	Coefficient	Show

Key questions

What's the relationship between the second difference and the coefficient of n^2?

What type of sequence is generated when comparing an^2 to the given quadratic sequence?

What are the steps in finding the n^{th} term of a quadratic sequence?

Exemplar Questions

Amir is investigating differences in quadratic sequences.

n^2	1	4	9	16
1st Difference	3	5	7	
2nd Difference	2	2		

Sequence	2nd difference	n^{th} term
1, 4, 9, 16	2	n^2
2, 8, 18, 32		$2n^2$
3, 12, 27, 48		
4, 16, 36, 64		

Follow Amir's method to complete the table. What do you notice about the relationship between the 2nd difference and the coefficient of n^2?

Mo is finding the n^{th} term of 1, 10, 23, 40

Copy and complete his steps.

Sequence				n^{th} term
2	8	18	32	$2n^2$
↓ -1	↓ 2	↓ ___	↓ ___	$3n$ _____
1	10	23	40	$2n^2+3n$_____

2nd difference is ___ so the coefficient of n^2 is ___

Show that the 20th term is 856

Find the n^{th} term of the following sequences.

- 2, 9, 22, 41
- 2, 16, 42, 80
- 1, 8, 21, 40
- 4, 18, 44, 82

Find the n^{th} term of the sequence -2, 9, 24, 43

Which term in the sequence has a value of 241?