

Non-Calculator Methods

Year 10

#MathsEveryoneCan

White  
Rose  
Maths

	Week 1	Week 2	Week 3	Week 4	Week 5	Week 6	Week 7	Week 8	Week 9	Week 10	Week 11	Week 12
<b>Autumn</b>	<b>Similarity</b>						<b>Developing Algebra</b>					
	Congruence, similarity and enlargement			Trigonometry			Representing solutions of equations and inequalities			Simultaneous equations		
<b>Spring</b>	<b>Geometry</b>						<b>Proportions and Proportional Change</b>					
	Angles & bearings		Working with circles		Vectors		Ratios & fractions		Percentages and Interest		Probability	
<b>Summer</b>	<b>Delving into data</b>				<b>Using number</b>				<b>Expressions</b>			
	Collecting, representing and interpreting data				Non-calculator methods		Types of number and sequences		Indices and Roots		Manipulating expressions	

# Summer 2: Using Number/Expressions

## Weeks 1 and 2: Types of Number and Sequences

This block again mainly revises KS3 content, reviewing prime factorisation and associated number content such as HCF and LCM. Sequences is extended for Higher Tier to include surds and finding the formula for a quadratic sequence.

National curriculum content covered:

- consolidating subject content from key stage 3:
  - factors, multiples, primes, HCF and LCM
  - describe and continue sequences
- recognise and use sequences of triangular, simple arithmetic progressions, Fibonacci type sequences, quadratic sequences, and simple geometric progressions ( $r^n$  where  $n$  is an integer, and  $r$  is a positive rational number **{or a surd}**) **{and other sequences}**
- deduce expressions to calculate the  $n$ th term of linear **{and quadratic}** sequence

## Weeks 3 and 4: Indices and roots

This block consolidates the previous two blocks focusing on understanding powers generally, and in particular in standard form. Negative and fractional indices are explored in detail. Again, much of this content will be familiar from KS3, particularly for Higher tier students, so this consolidation material may be covered in less than two weeks allowing more time for general non-calculator and problem-solving practice. To consolidate the index laws, these can be revisited in the next block when simplifying algebraic expressions.

National curriculum content covered:

- recognise and use sequences of square and cube numbers
- **{estimate powers and roots of any given positive number}**
- calculate with roots, and with integer **{and fractional}** indices
- calculate with numbers in standard form  $A \times 10^n$ , where  $1 \leq A < 10$  and  $n$  is an integer
- simplifying expressions involving sums, products and powers, including the laws of indices

## Weeks 5 and 6: Manipulating expressions

This final block of year 10 builds on the Autumn term learning of equations and inequalities, providing revision and reinforcement for Foundation tier students and an introduction to algebraic fractions for those following the Higher tier. This also allows all students to revise fraction arithmetic to keep their skills sharp. Algebraic argument and proof are considered, starting with identities and moving on to consider generalised number.

National curriculum content covered:

- simplify and manipulate algebraic expressions (including those involving surds **{and algebraic fractions}**) by factorising quadratic expressions of the form  $x^2 + bx + c$
- know the difference between an equation and an identity
- argue mathematically to show algebraic expressions are equivalent, and use algebra to support and construct arguments **{and proofs}**

## Why Small Steps?

We know that breaking the curriculum down into small manageable steps should help students to understand concepts better. Too often, we have noticed that teachers will try and cover too many concepts at once and this can lead to cognitive overload. We believe it is better to follow a “small steps” approach.

As a result, for each block of content in the scheme of learning we will provide a “small step” breakdown. ***It is not the intention that each small step should last a lesson – some will be a short step within a lesson, some will take longer than a lesson.*** We would encourage teachers to spend the appropriate amount of time on each step for their group, and to teach some of the steps alongside each other if necessary.

## What We Provide

- Some ***brief guidance*** notes to help identify key teaching and learning points
- A list of ***key vocabulary*** that we would expect teachers to draw to students’ attention when teaching the small step,
- A series of ***key questions*** to incorporate in lessons to aid mathematical thinking.
- A set of questions to help ***exemplify*** the small step concept that needs to be focussed on.

Year 10 | Autumn Term 1 | Congruence, Similarity and Enlargement

### Identify similar shapes

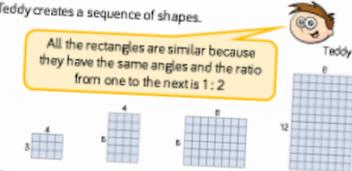
**Notes and guidance**  
Students apply their knowledge of enlargement to identifying similar shapes. They should relate the ideas of proportionate corresponding lengths and the fact that the angles remain unchanged. It is helpful for students to understand ratio within this context as this will be useful later when introduced to trigonometry in the following block of small steps.

**Key vocabulary**

Enlarge	Scale factor	Ratio
Similar	Proportion	

**Exemplar Questions**  
Teddy creates a sequence of shapes.

All the rectangles are similar because they have the same angles and the ratio from one to the next is 1:2

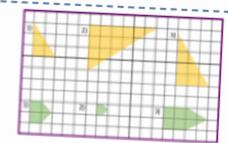


Do you agree with Teddy's statement? Which rectangles are similar? Justify your answer.

Explain how you know these shapes are similar.



Decide which shapes in each group are similar. Explain why you think they are or are not similar.



**Key questions**

How can you confirm that two shapes are similar?

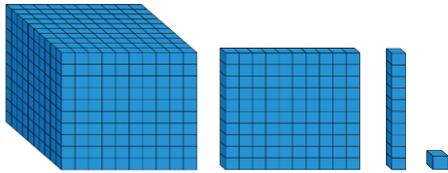
How can you use ratio to show that two shapes are/are not mathematically similar?

What do you notice about the angles of similar shapes?

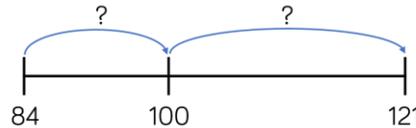
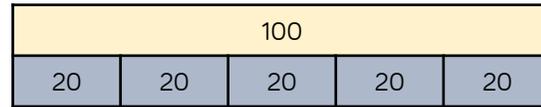
- These include reasoning and problem-solving questions that are fully integrated into the scheme of learning. Depending on the attainment of your students, you may wish to use some or all of these exemplars, which are in approximate order of difficulty. Particularly challenging questions are indicated with the symbol .
- For each block, we also provide ideas for key representations that will be useful for all students.

Some of the small steps are in **bold** and labelled with **H** to indicate this is Higher tier GCSE content. We would encourage teachers to use these with as many students as possible – if you feel your class can access any particular small step, then please include it in your planning. Steps that review content covered at Key Stage 3 are labelled **R**.

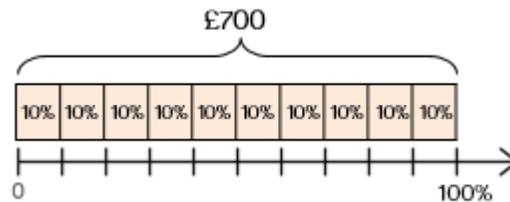
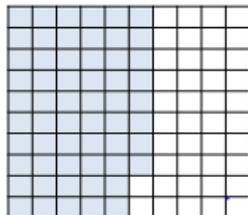
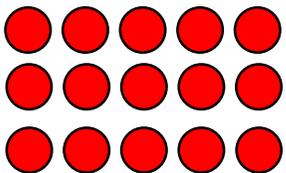
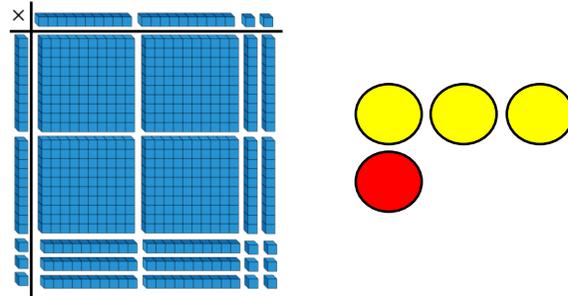
# Key Representations



£600



	Hundreds	Tens	Ones
	?		
+			
		?	?



Although many students will be confident with non-calculator methods by Year 10, those who have not developed conceptual understanding will still benefit from the use of manipulatives to support the methods for the four operations of number. Many will find double-sided counters useful to support calculations involving directed number.

For all students, bar models and/or number lines will provide a “way in” to the problem solving questions in this block, helping them to decide whether an additive or multiplicative approach is appropriate, and the choice of operation.

# Non-calculator methods

## Small Steps

- ▶ Mental/written methods of integer/decimal addition and subtraction R
- ▶ Mental/written methods of integer/decimal multiplication and division R
- ▶ The four rules of fraction arithmetic R
- ▶ Exact answers
- ▶ **Rational and irrational numbers (convert recurring decimals here)** H
- ▶ **Understand and use surds** H
- ▶ **Calculate with surds** H
- ▶ Rounding to decimal places and significant figures R

H denotes Higher Tier GCSE content

R denotes 'review step' - content should have been covered at KS3

# Non-calculator methods

## Small Steps

- ▶ Estimating answers to calculations R
- ▶ Understand and use limits of accuracy
- ▶ **Upper and lower bounds** H
- ▶ Use number sense
- ▶ Solve financial maths problems
- ▶ Break down and solve multi-step problems

H denotes Higher Tier GCSE content

R denotes 'review step' – content should have been covered at KS3

# Addition and Subtraction



## Notes and guidance

This step looks over mental and written methods for addition and subtraction. Teachers could use this to revisit the skills in the contexts their class need to particular revise e.g. calculating angles, financial maths problems etc. Similarly, this step need not be covered as a one-off, but the content could be covered in starters or in other relevant lessons. It is useful to keep the skills covered sharp by revisiting regularly for the remainder of the course.

## Key vocabulary

Add	Subtract	Balance
Adjust	Credit/Debit	Profit/Loss

## Key questions

What strategies do you know to add/subtract numbers mentally?

Why is easy to add (e.g.) 99/9.9 etc. without a written method?

How do we set up written methods for addition and subtraction? What might go wrong with decimals?

## Exemplar Questions

Explain how each of these calculations can be done mentally. Compare your methods with a partner's.

$142 + 99$

$£20 - £7.68$

$180 - 62$

$360 - 148$

$- 7 - 12$

$658 - 299$

$25 - 40$

$4.3 + 14.9$

Complete the bank statement.

Date	Description	Credit (£)	Debit (£)	Balance (£)
Jun 1	Opening balance			147.52
Jun 3	Phone bill		38.65	
Jun 4	Wages	208.85		
Jun 8	Rent			171.82

Cinema Tickets	
Adult	£8.90
Child	£4.65

- ♦ Dora buys one adult ticket and one child ticket. How much change should she receive from £20?
- ♦ Brett buys an adult ticket. He has a 10% student discount. How much does Brett pay for his ticket?

# Multiplication and Division



## Notes and guidance

Building on the previous step, students can focus on multiplication and division whilst considering problems involving all four operations. Special care needs to be taken with decimals e.g.  $6 \div 0.2 = 60 \div 2$  but  $3.6 \times 1.7 \neq 36 \times 17$ . Teachers can again choose which skills to revisit e.g. area, substitution etc. and likewise keep the skills covered sharp by revisiting regularly for the remainder of the course.

## Key vocabulary

Multiply	Divide	Adjust
Perimeter	Volume	Area

## Key questions

- What strategies do you know to multiply numbers mentally?
- How do we set up long multiplication/division?
- How can we adjust calculations if the numbers involve decimals? Is it the same or different for multiplication and division?

## Exemplar Questions

Explain how each of these calculations can be done mentally. Compare your methods with a partner's.

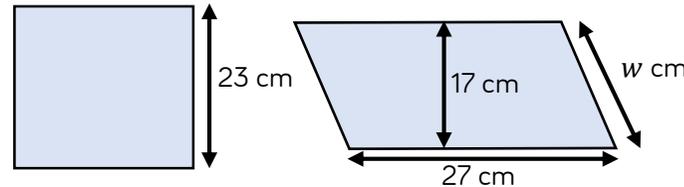
$$8 \times 0.04$$

$$0.48 \times 1000$$

$$18 \div 0.3$$

$$37 \times 99$$

The perimeters of the square and parallelogram are equal.



- Find the value of  $w$ .
- Work out the difference between the areas of the shapes.

Work out the calculations on the cards.

Compare your strategies with a partner's.

$$36 \times 74$$

$$0.36 \times 7.4$$

$$89.1 \div 6$$

$$360 \div 24$$

Solve the problems, showing all your working clearly.

- A calculator costs £4.79. Find the cost of 38 of these calculators.
- A two-week holiday costs £1659. How much does the holiday cost per day?
- A metal alloy contains zinc, copper and nickel in the ratio 2 : 3 : 7. How much nickel is there in 702 g of the alloy?
- Work out the volume of a cube of side 4.5 cm

# Fraction arithmetic



## Notes and guidance

Students often confuse the rules of fraction arithmetic so it is worth revisiting the similarities and differences when working with the different operations. They have, however, been working with these for several years so at this stage it is useful to include context in practice questions whenever possible. Bar models and other pictorial support are still useful and should be encouraged, both to support choice of operation and to help to reason how to perform the calculation.

## Key vocabulary

Fraction	Numerator	Denominator
Reciprocal	Mixed number	Improper fraction

## Key questions

Is working with mixed numbers different from working with fractions when adding/multiplying etc.?

Can you draw a picture to show how fraction multiplication works?

How is fraction multiplication different from fraction division?

## Exemplar Questions

Work out the calculations on the cards and put your answers in order of size, starting with the smallest.

$$\frac{3}{5} - \frac{1}{2}$$

$$\frac{3}{5} \times \frac{1}{2}$$

$$\frac{3}{5} \div \frac{1}{2}$$

$$\frac{3}{5} + \frac{1}{2}$$

A company employs 160 people.

$\frac{5}{8}$  of the employees are women.

$\frac{2}{5}$  of the women work part time.

$\frac{5}{12}$  of the men work part time.

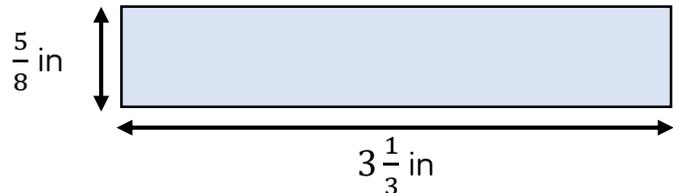
Work out the total number of people who work part time.

Work out which TV is cheaper.

Was £1800  
Now  $\frac{1}{3}$  off

Was £1600  
Now 20% off

Find the perimeter and the area of the rectangle.



## Exact answers

### Notes and guidance

This step prepares Higher tier students for the upcoming study of surds as well as reminding all of language such as “in terms of  $\pi$ ” etc. Use of formulae for area/perimeter/volume of shapes involving circles can be revisited here and it is useful to involve “reverse questions” such as finding the height of a cone given its volume and radius. It is also a timely opportunity to revisit exact trigonometrical values met in the Autumn term.

### Key vocabulary

Exact	In terms of	Square/Cube Root
Sine	Cosine	Tangent

### Key questions

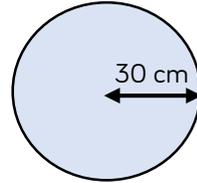
Is it okay to give the solution to an equation as a fraction rather than a decimal? Why or why not?

Can you check exact answers for (e.g.) volume of a cylinder on a calculator? What mode do you need?

What’s the difference between the sine, cosine and tangent of an angle?

### Exemplar Questions

Which card shows the circumference of the circle?

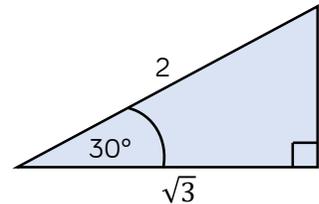
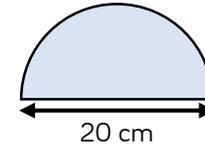


$30\pi$  cm

$60\pi$  cm

$900\pi$  cm

Find the area of the semicircle.  
Give your answer in terms of  $\pi$ .

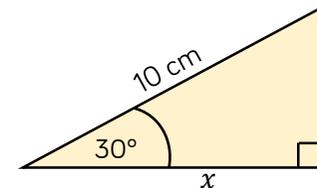


Use the diagram to write down the exact values of:

- $\sin 30^\circ$
- $\tan 30^\circ$
- $\cos 30^\circ$
- $\sin 60^\circ$



Work out the value of  $x$ .



Match the equations to the answers.

$2x^2 = 14$

$3a + 5 = 7$

$2b - 1 = 2$

$12 = 5 + 3t$

$\frac{7}{3}$

$\sqrt{7}$

$\frac{3}{2}$

$\frac{2}{3}$

# Rational/Irrational Numbers



## Notes and guidance

Students have already met irrational numbers such as  $\pi$  and  $\sqrt{2}$  and this step formalises this learning and the associated language.  $\pi$  is useful for generating other irrationals e.g.  $2\pi, \pi + 3$  are both irrationals between 6 and 7. Students will learn that recurring decimals are not irrational and how to convert these to fractions. Using this technique to show that  $0.\dot{9} = 1$  (or  $3 \times 0.\dot{3} = 3 \times \frac{1}{3}$ ) is an interesting extension.

## Key vocabulary

Integer	Decimal	Terminating
Recurring	Infinite	Root

## Key questions

- Do all linear equations have rational solutions? Why or why not?
- Which of the exact trigonometric values are rational, and which are irrational?
- What's the difference between a terminating decimal and a recurring decimal?

## Exemplar Questions

Which of these equations have rational solutions?

$3x = 1$	$x^2 = 1$	$x^2 = 3$	$4x^2 = 1$
$4x = \pi$	$x^2 - 3 = 1$	$x^2 = 64$	$x^3 = 64$

Complete the workings to find  $0.\dot{7}$  as a fraction.

Let  $x = 0.\dot{7}$   
 So  $10x = 7.\dot{7}$   
 So  $9x = \dots$   
 So  $x = \dots$

What would be the same and what would be different when writing these recurring decimals as fractions?

$0.\dot{3}$     
   $0.0\dot{3}$     
   $0.1\dot{3}$     
   $0.\dot{1}3$     
   $1.\dot{3}$

Decide whether these statements are always, sometimes or never true, justifying your answers.

Terminating decimals are rational	Recurring decimals are irrational
Multiples of $\pi$ are irrational	$\pi + a$ is irrational
Square roots of fractions are irrational	Cube roots of fractions are irrational

# Understand and use surds



## Notes and guidance

This first of two steps on surds, which could be introduced concurrently, looks at the definition of a surd as the irrational root of a rational number, and writing surds in simplified form. When simplifying e.g.  $\sqrt{72}$  it is useful to compare using 36 and 2 as a factor pair rather than 8 and 9, demonstrating that they both lead to the same final answer but that finding the largest square factor is much more efficient.

## Key vocabulary

Surd	Square root	Cube root
Simplify	Factors	

## Key questions

- How can you tell if the square root of an integer less than 100 will be a surd or not?
- Are surds rational or irrational?
- What are the factor pairs of \_\_\_? Which factor pair has the largest square factor?
- Can you use your calculator to check surd simplification?

## Exemplar Questions

Work out the calculations on the cards, using a calculator where necessary, giving your answers as integers, fractions or mixed numbers.

$\sqrt{36}$

$\sqrt{\frac{4}{9}}$

$\sqrt{7} \times \sqrt{7}$

$\sqrt{6\frac{1}{4}}$

$\sqrt{196}$

$(\sqrt{11})^2$

$(\sqrt{15})^2$

$(\frac{\sqrt{3}}{2})^2$

Use a calculator to match the cards that are of equal value.

$\sqrt{40} \div \sqrt{2}$

$\sqrt{10} \times \sqrt{3}$

$\sqrt{4} \times \sqrt{6}$

$\sqrt{20}$

$\sqrt{2} \times \sqrt{3}$

$\sqrt{30}$

$\sqrt{5}$

$\sqrt{50} \div \sqrt{10}$

$\sqrt{6}$

$\sqrt{24}$

Are these generalisations true or false?

$\sqrt{ab} = \sqrt{a} \times \sqrt{b}$

$\sqrt{a \div b} = \sqrt{a} \div \sqrt{b}$



$$\begin{aligned} \sqrt{50} &= \sqrt{25} \times \sqrt{2} \\ &= 5\sqrt{2} \end{aligned}$$

Use Ron's method to write the surds in the form  $a\sqrt{b}$

- ♦  $\sqrt{75}$
- ♦  $\sqrt{27}$
- ♦  $\sqrt{98}$
- ♦  $\sqrt{63}$

Is it better to simplify  $\sqrt{12}$  using  $12 = 2 \times 6$  or  $12 = 4 \times 3$ ? Why?

# Calculate with surds



## Notes and guidance

Having established the behaviour of surds when multiplied and divided in the previous step, we now investigate addition and subtraction. Students can establish rules for themselves, using calculators in both exact and decimal forms.

Sometimes students are confused that although  $\sqrt{a} + \sqrt{b} = \sqrt{a+b}$  it is possible to simplify e.g.  $8\sqrt{2} - 3\sqrt{2}$ . It is useful to compare this to e.g.  $8x - 3x$  or  $8\pi - 3\pi$ . Rationalising the denominator is also explored.

## Key vocabulary

Surd	Square root	Cube root
Simplify	Rationalise	Denominator

## Key questions

When is it possible to simplify surd expressions involving addition and subtraction, and when is it not possible?

What's the first step when rationalising a denominator?

When expanding brackets involving surds, why might we sometimes get rational terms?

## Exemplar Questions

Which of the calculations are correct and which are incorrect?

❖  $\sqrt{2} + \sqrt{3} = \sqrt{5}$

❖  $\sqrt{8} - \sqrt{3} = \sqrt{5}$

❖  $6\sqrt{3} - \sqrt{3} = 6$

❖  $5\sqrt{2} + 2\sqrt{2} = 7\sqrt{2}$



Whitney

$$\sqrt{75} - \sqrt{27} = \sqrt{48}$$

$$\begin{aligned} \sqrt{75} - \sqrt{27} &= 5\sqrt{3} - 3\sqrt{3} \\ &= 2\sqrt{3} \end{aligned}$$



Amir

❖ Who do you agree with?

❖ Work out  $\sqrt{98} - \sqrt{50}$

Complete the steps to write  $\frac{5}{2\sqrt{3}}$  as a fraction with a rational denominator.

$$\frac{5}{2\sqrt{3}} = \frac{5}{2\sqrt{3}} \times \frac{\sqrt{3}}{\sqrt{3}} = \frac{\square\sqrt{3}}{2 \times \square} = \frac{\square\sqrt{3}}{\square}$$

Complete the calculation.

$$\begin{aligned} (2 + \sqrt{3})^2 &= (2 + \sqrt{3})(2 + \sqrt{3}) \\ &= 4 + \square + \square + 3 \\ &= 7 + \square\sqrt{3} \end{aligned}$$



Work out  $(2\sqrt{3} - \sqrt{2})(2\sqrt{3} + \sqrt{2})$      $(3 - \sqrt{3})^3$

# Rounding



## Notes and guidance

Rounding could be required in both calculator and non-calculator papers, so although this block as a whole is focusing on the latter, it may be appropriate to use calculators for some of the questions in this step. Again, this is largely revision and can be done in the context of other areas of mathematics if appropriate, perhaps reminding students of the difference between decimal places and significant figures first if needed.

## Key vocabulary

Degree of Accuracy	Decimal place
Round	Approximate
	Significant Figure

## Key questions

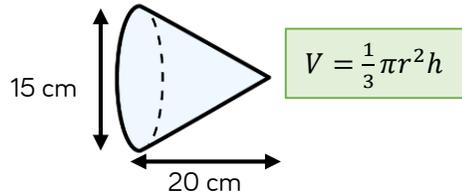
What's the difference between decimal places and significant figures?

How do you decide what degree of accuracy to give an answer to? Is the full calculator display always, sometimes or never appropriate?

## Exemplar Questions

Find the volume of the cone, giving your answer

- in terms of  $\pi$
- correct to the nearest integer
- correct to 1 decimal place
- correct to 3 significant figures



Which degree of accuracy do you think is the most appropriate?

Write each of these numbers to two significant figures.

0.0385	0.385	3.85	38.5	385
600.123	609.123	0.609123	0.009123	

Work out  $\frac{38.7 + 1.5^3}{9.2}$

Write down all the figures on your calculator display. Write your answer correct to 2 decimal places.

12 357 people attended a football match.

- Write 12 357 in words
- Write 12 357 correct to the nearest thousand
- Write 12 357 correct to three significant figures

# Estimating



## Notes and guidance

Although the focus of this step will be rounding to one significant figure to support estimation, it is worth including examples where using factors might be more useful e.g. 12 on a denominator if the numerator involves multiplying 30 and 40. Students should also use their knowledge of square (and possibly cube numbers) to estimate roots. Students could also revisit fraction arithmetic by using  $\pi \approx \frac{22}{7}$  in circle calculations.

## Key vocabulary

Round	Approximate	Decimal place
Degree of Accuracy		Significant Figure

## Key questions

Why do you need to be careful when rounding decimals when making estimates?

How would you estimate (e.g.) 38% of \_\_\_?

Between which two integers does the square root of \_\_\_ lie? How do you know?

## Exemplar Questions

By rounding each number to 1 significant figure, estimate the answers to the calculations.

$$86 \times 77$$

$$8.6 \times 7.6$$

$$0.86 \times 7.5$$

$$0.86 \times 0.74$$

Can you tell which estimates are overestimates and which are underestimates?

Annie is estimating the area of a circle of diameter 40 cm.

$$\begin{aligned} 3 \times 20^2 \\ = 60^2 \\ = 3600 \end{aligned}$$



Is her estimate a good one? Justify your answer.

What mistake has been made in this estimate?

$$830 \div 0.37 = \frac{830}{0.37} \approx \frac{800}{0.4} = \frac{80}{4} = 20$$



Estimate the answer to:  $\frac{27 \times 35}{0.056}$

$$0.19^2$$

$$\sqrt{57} \approx 8.4$$



Explain why Mo must be wrong.

Estimate  $\sqrt{83}$

$\sqrt{24}$

$\sqrt[3]{30\,000}$

$$\frac{31.9 + \sqrt{97.8}}{0.191}$$

$$\frac{\sqrt{907}}{11.6}$$

$$\sqrt{\frac{600}{14.8 + 4.7^2}}$$

# Limits of accuracy

## Notes and guidance

Students have met error intervals at KS3 and this step takes this a little further to include the effect of truncation as well as rounding. In both cases it is worth using a number line to illustrate e.g. if a number has been rounded to 4.6 to 1 decimal place then using a line from 4.5 to 4.7 helps students to see where the limits are, contrasting this with truncation (shortening) meaning the original number cannot possibly be below 4.6

## Key vocabulary

Limit	Error interval	Upper/Lower bound
Truncate	Round	Correct to...

## Key questions

What numbers might round to give (e.g.) 4.6 to 1 decimal place? Can you think of examples both below and above 4.6? What would the limits/bounds be? Show me on a number line.

What numbers might be truncated to give 4.6 to 1 decimal place? Why is 4.599 not a possible value?

## Exemplar Questions

The number of people who attended a concert is 20 000, correct to the nearest thousand.

- What is the smallest number of people who could have been at the concert?
- What is the greatest number of people who could have been at the concert?
- How would your answers change if the attendance figure had been correct to the nearest hundred?

A length  $l$  is given as 7.4 cm correct to one decimal place.

Complete the error interval for the length  $l$ .

$$\underline{\quad} \leq l < \underline{\quad}$$

Complete the error interval for the length  $l$  if it had been **truncated** to one decimal place.

$$\underline{\quad} \leq l < \underline{\quad}$$

Find the upper and lower bounds of the numbers given to the degrees of accuracy shown.

£7 rounded to the nearest pound

7 rounded to the nearest integer

7.0 rounded to one decimal place

70 rounded to the nearest 10

70 rounded to the nearest integer

7.00 rounded to two decimal places

7.00 truncated to two decimal places

# Upper and lower bounds



## Notes and guidance

This builds on the previous step using the limits of accuracy to find upper and lower bounds of calculations. Students often find this relatively straightforward for multiplication and addition as e.g.  $(a + b)_{\max} = a_{\max} + b_{\max}$  is quite intuitive, but subtraction and division need more careful exploration to establish e.g.  $(a - b)_{\max} = a_{\max} - b_{\min}$ . Learning such formulas is not recommended here, just considering “How could we make the value greater/smaller?”

## Key vocabulary

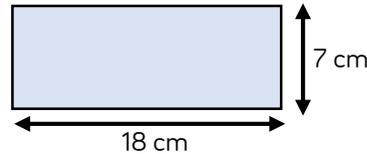
Upper/lower bound	Maximum/minimum		
Sum	Difference	Product	Quotient

## Key questions

If we want (e.g.)  $ab$  to be as large as possible, do the values of  $a$  and  $b$  also need to be as large as possible? Is it the same or is it different for  $\frac{a}{b}$  to be as large a possible? In a formula such as  $V = \frac{1}{3}\pi r^2 h$ , which components don't vary when looking for the minimum value of  $V$ ?

## Exemplar Questions

The dimensions of the rectangle are given to the nearest cm.



Write, the upper and lower bounds of:

- the length of the rectangle
- the width of the rectangle
- the perimeter of the rectangle
- the area of the rectangle

Give your answers to 2 decimal places.

$x = 7.4$  and  $y = 3.5$ , both to 1 decimal place.

Dexter is wrong.  
Show that the upper bound of  $x - y$  is 4

The upper bound of  $x - y$  is  $7.45 - 3.55 = 3.9$



Find the upper bounds of  $\frac{x}{y}$        $\frac{y}{x}$

Find the lower bounds of  $y + 2x$        $y^2 - x$

Aisha runs 100 m in 15 seconds.

Find the upper and lower bounds of Aisha's speed if the numbers are given  $\blacksquare$  to the nearest integer  $\blacksquare$  to two significant figures  $\blacksquare$

$F = 32.87$  N correct to 2 decimal places.

$A = 5.16$  m<sup>2</sup> correct to 3 significant figures.

Work out the value of  $p$  to a suitable degree of accuracy.  
Explain your choice.

$$p = \frac{F}{A}$$

## Use number sense

### Notes and guidance

Building on the strategies used for the four operations in the earlier review steps, this step focuses on deriving facts from known facts. Students often struggle to decide whether e.g.  $39 \times 73 = 39 \times 72 + 39$  or  $39 \times 72 + 72$ ; area models are very useful to illustrate this. Students could also be taught to look for factor pairs such as 2 and 5, 4 and 25, 8 and 125 to make multiplications easier, and factorising to simplify division e.g. divide by 2 and then by 7 instead of dividing by 14

### Key vocabulary

Adjust      Compensate      Factorise

### Key questions

What are useful pairs of factors to look for in order to simplify a calculation?

If we know the value of an algebraic expression (e.g.  $4xy$ ), what other expressions can we work out very easily?

How can we adjust (e.g.  $9.9 \times 87$ ) to make it into easier calculations?

### Exemplar Questions

Use the fact that  $39 \times 72 = 2808$  to find the values of the cards. Explain how you found your answers.

$$2808 \div 72$$

$$2808 \div 7.2$$

$$28.08 \div 7.2$$

$$39 \times 36$$

$$39 \times 73$$

$$37 \times 72$$

35% of a number is 112

What other percentages of the number can you easily find?

Work out  $\frac{5}{8}$  of the number       $\frac{11}{8}$  of the number

$$\begin{aligned} 75 \times 36 & \\ = 25 \times 3 \times 4 \times 9 & \\ = 25 \times 4 \times 3 \times 9 & \\ = 100 \times 12 & \\ = 1200 & \end{aligned}$$

Use factorisation or other strategies to work out:

$$14 \times 45$$

$$7665 \div 35$$

$$175 \times 28$$

$$19\,248 \div 48$$

$$9.9 \times 87$$

$$13\,692 \div 42$$

Without calculating the answers, explain how you know:

$$96 \div 2 > 96 \div 3$$

$$\frac{4}{7} \text{ of } 420 > 200$$

$$\frac{7}{12} \text{ of } 240 > \frac{11}{20} \times 240$$

$$\frac{7}{15} \text{ of } \pounds 600 < 55\% \text{ of } \pounds 600$$

$$45\% \text{ of } 80 = 80\% \text{ of } 45$$

# Financial Maths

## Notes and guidance

Here students can revise the language and methods studied in previous units, notably within Percentages and Interest. As well as the chance to revisit compound and simple interest, students could explore personal finance such as the tax system, and utility bills. Many financial organisations provide materials to support this. As an extension, students could look at more complex products such as mortgages or investigating the meaning of terms like APR.

## Key vocabulary

Credit/Debit	Profit/Loss	VAT
Standing Charge	Allowance	Tax

## Key questions

What is the first step you need to take to solve the problem?

Which words tell you if you need to add/subtract/multiply or divide?

## Exemplar Questions

Kim gets paid £8.20 an hour for the first 35 hours she works in a week, and any additional hours she works are paid at one and a half times her normal hourly rate.

One week, Kim works eight hours every day from Monday to Saturday.

▣ Calculate Kim's total wage for this week.

Kim pays 32% of her wage in taxes and other deductions.

▣ Calculate Kim's wage after these deductions.

Alex buys a house for £140 000

She spends £40 000 renovating the house.

She then sells the house for £235 000

Work out Alex's percentage profit on the sale of the house.

Pervious meter reading	3672
Current meter reading	4218
Units used	
Standing charge	£17.50
Total charges	
VAT at 5%	
Total to pay	

The table shows part of a gas bill. The cost of each unit of gas is 84.3p. Complete the table.

Esther earns £28 000 a year.

She pays 20% tax on earnings over £12 500

She pays 12% National Insurance on earnings over £8632

Work out Esther's monthly take-home salary.

# Multi-step problems

## Notes and guidance

Students often find it difficult to access GCSE questions that require several steps of working. Students can be prepared for these by practising questions that require an increasing number of steps and complexity. This is another opportunity to revisit and remind students of formulae such as those for area, volume, pressure and density, but the focus should be on looking at what information is given and what can be found out as students work towards a solution.

## Key vocabulary

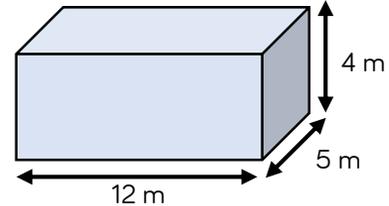
Force	Pressure	Area
Density	Mass	Volume

## Key questions

- What information do we need to solve the problem?
- What can we find out first? Given this new information, what can we find out next?
- What formulas do we need to know to tackle this problem?

## Exemplar Questions

The diagram shows a shipping container on horizontal ground. The weight of the container is 24 000 N. Use the formula  $p = \frac{F}{A}$  to work out the pressure exerted on the ground by the shipping container.

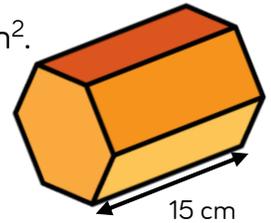


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Cat food comes in 825 g packets that cost £2.15  
 Huan has two cats.  
 Each cat eats 55g of cat food a day.  
 Huan wants to buy enough cat food for six weeks.  
 How much will the cat food cost?

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The diagram shows a hexagonal prism. The area of the cross-section of the prism is 80 cm<sup>2</sup>. The density of the prism is 25 g/cm<sup>3</sup>. 1 kg of the material used to make the prism costs £650  
 Find the cost of the prism.



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A water tank holds 120 litres of water. There is a hole in the water tank and water leaks out at a rate of 80 millilitres per second. Work out the time it takes for the water tank to empty completely. Give your answer in minutes.