Reasoning and Problem Solving

Percentages

Spring - Block 2
Overview

Small Steps

- Understand percentages
- Fractions to percentages
- Equivalent FDP
- Order FDP
- Percentage of an amount (1)
- Percentage of an amount (2)
- Percentages – missing values

Notes for 2020/21

Children should have been introduced to percentages briefly in Y5 but this work may have been missed. Time spent exploring 100 as a denominator, making the link to decimals and hundredths is important. Bar models and hundred squares should be used to support understanding.
Children are introduced to ‘per cent’ for the first time and will understand that ‘per cent’ relates to ‘number of parts per hundred’.

They will explore this through different representations which show different parts of a hundred. Children will use ‘number of parts per hundred’ alongside the % symbol.

There are ____ parts per hundred shaded. This is ____%.

<table>
<thead>
<tr>
<th>Pictorial</th>
<th>Parts per hundred</th>
<th>Percentage</th>
</tr>
</thead>
<tbody>
<tr>
<td><img src="image" alt="Pictorial" /></td>
<td><img src="image" alt="Parts per hundred" /></td>
<td>75%</td>
</tr>
</tbody>
</table>

How many parts is the square split in to?

How many parts per hundred are shaded/not shaded?

Can we represent this percentage differently?

Look at the bar model, how many parts is it split into?

If the bar is worth 100%, what is each part worth?

Complete the sentence stem for each diagram.

Complete the table.

Complete the bar models.
Oh no! Dexter has spilt ink on his hundred square.

Complete the sentence stems to describe what percentage is shaded.

It could be...

It must be...

It can’t be...

Some possible answers:

It could be 25%

It must be less than 70%

It can’t be 100%

Mo, Annie and Tommy all did a test with 100 questions. Tommy got 6 fewer questions correct than Mo.

Complete the table. How many more marks did each child need to score 100%?

<table>
<thead>
<tr>
<th>Name</th>
<th>Score</th>
<th>Percentage</th>
</tr>
</thead>
<tbody>
<tr>
<td>Mo</td>
<td>56 out of 100</td>
<td>56%</td>
</tr>
<tr>
<td>Annie</td>
<td></td>
<td>65%</td>
</tr>
<tr>
<td>Tommy</td>
<td></td>
<td>50%</td>
</tr>
</tbody>
</table>

Mo needs 44
Annie needs 35
Tommy needs 50

Dora and Amir each have 100 sweets. Dora eats 65% of hers. Amir has 35 sweets left. Who has more sweets left?

56%  
65 out of 100  
50 out of 100  
50%

Neither. They both have an equal number of sweets remaining.
Notes and Guidance

It is important that children understand that ‘percent’ means ‘out of 100’.
Children will be familiar with converting some common fractions from their work in Year 5.
They learn to convert fractions to equivalent fractions where the denominator is 100 in order to find the percentage equivalent.

Mathematical Talk

What does the word ‘percent’ mean?

How can you convert tenths to hundredths?

Why is it easy to convert fiftieths to hundredths?

What other fractions are easy to convert to percentages?

Fractions to Percentages

What fraction of each hundred square is shaded?
Write the fractions as percentages.

Complete the table.

<table>
<thead>
<tr>
<th>Fraction</th>
<th>Percentage</th>
</tr>
</thead>
<tbody>
<tr>
<td>1/2</td>
<td></td>
</tr>
<tr>
<td>1/4</td>
<td></td>
</tr>
<tr>
<td>1/10</td>
<td></td>
</tr>
<tr>
<td>1/5</td>
<td></td>
</tr>
</tbody>
</table>

Fill in the missing numbers.

\[
\frac{12}{100} = \frac{\Box}{100} = \Box\% \quad \frac{12}{50} = \frac{\Box}{100} = \Box\% \\
44 = \frac{22}{100} = 22\%
\]
Reasoning and Problem Solving

In a Maths test, Tommy answered 62% of the questions correctly.

Rosie answered \( \frac{3}{5} \) of the questions correctly.

Who answered more questions correctly?

Explain your answer.

Tommy answered more questions correctly because \( \frac{3}{5} \) as a percentage is 60% and this is less than 62%.

Amir thinks that 18% of the grid has been shaded.

Dora thinks that 36% of the grid has been shaded.

Who do you agree with?

Explain your reasoning.

Dora is correct because \( \frac{18}{50} = \frac{36}{100} \).
Equivalent FDP

Notes and Guidance

Children use their knowledge of common equivalent fractions and decimals to find the equivalent percentage.

A common misconception is that 0.1 is equivalent to 1%. Diagrams may be useful to support understanding the difference between tenths and hundredths and their equivalent percentages.

Mathematical Talk

How does converting a decimal to a fraction help us to convert it to a percentage?

How do you convert a percentage to a decimal?

Can you use a hundred square to represent your conversions?

Varied Fluency

Complete the table.

<table>
<thead>
<tr>
<th>Decimal</th>
<th>Fraction</th>
<th>Percentage</th>
</tr>
</thead>
<tbody>
<tr>
<td>0.35</td>
<td>$\frac{35}{100}$</td>
<td>35%</td>
</tr>
<tr>
<td>0.27</td>
<td></td>
<td></td>
</tr>
<tr>
<td>0.6</td>
<td></td>
<td></td>
</tr>
<tr>
<td>0.06</td>
<td></td>
<td></td>
</tr>
</tbody>
</table>

Use $<$, $>$ or $=$ to complete the statements.

$$0.36 \phantom{0} 40\% \phantom{0} \frac{7}{10} \phantom{0} 0.07$$

$$0.4 \phantom{0} 25\% \phantom{0} 0.4 \phantom{0} \frac{1}{4}$$

Which of these are equivalent to 60%?

$\frac{60}{100}$, $\frac{6}{100}$, 0.06, $\frac{3}{5}$, $\frac{3}{50}$, 0.6
### Reasoning and Problem Solving

**Amir says 0.3 is less than 12% because 3 is less than 12**

**Explain why Amir is wrong.**

Amir is wrong because 0.3 is equivalent to 30%.

**Complete the part-whole model. How many different ways can you complete it?**

- **A = 0.3, 30% or \( \frac{3}{10} \)**
- **B = 0.2, 20%, \( \frac{2}{10} \) or \( \frac{1}{5} \)**
- **C = 0.1, 10% or \( \frac{1}{10} \)**

**Can you create your own version with different values?**

**How many different fractions can you make using the digit cards?**

- **Possible answers:**
  - Children make a range of fractions.
  - They should be able to convert 1, 2, 3, 1/2, 3/4, 4/5 into decimals and percentages.

**How many of the fractions can you convert into decimals and percentages?**
Children convert between fractions, decimals and percentages to enable them to order and compare them.

Encourage them to convert each number to the same form so that they can be more easily ordered and compared. Once the children have compared the numbers, they will need to put them back into the original form to answer the question.

What do you notice about the fractions, decimals or percentages? Can you compare any straight away?

What is the most efficient way to order them?

Do you prefer to convert your numbers to decimals, fractions or percentages? Why?

If you put them in ascending order, what will it look like?

If you put them in descending order, what will it look like?

Four friends share a pizza. Whitney eats 35% of the pizza, Teddy eats 0.4 of the pizza, Dora eats 12.5% of the pizza and Alex eats 0.125 of the pizza.

Write the amount each child eats as a fraction. Who eats the most? Who eats the least? Is there any left?

Use <, > or = to complete the statements:

| 60% | 0.6 | \( \frac{3}{5} \) |
| 0.23 | 24% | \( \frac{1}{4} \) |
| 37.6% | \( \frac{3}{8} \) | 0.27 |

Order from smallest to largest:

| 50% | \( \frac{2}{5} \) | 0.45 | \( \frac{3}{10} \) | 54% | 0.05 |
In his first Geography test, Mo scored 38%.
In the next test he scored 40%.

Did Mo improve his score?

Explain your answer.

Mo improved his score.
\[\frac{16}{40}\] is equivalent to 40% which is greater than his previous score of 38%.

Which month did Eva save the most money?

Estimate your answer using your knowledge of fractions, decimals and percentages.

Explain why you have chosen that month.

In January, Eva saves \(\frac{3}{5}\) of her £20 pocket money.

In February, she saves 0.4 of her £10 pocket money.

In March, she saves 45% of her £40 pocket money.

She saved the most money in March.

Estimates:
Over £10 in January because \(\frac{3}{5}\) is more than half.
Under £10 in February because she only had £10 to start with and 0.4 is less than half.
Nearly £20 in March because 45% is close to a half.
Notes and Guidance

Children use known fractional equivalences to find percentages of amounts. Bar models and other visual representations may be useful in supporting this e.g. 25% = \(\frac{1}{4}\) so we divide into 4 equal parts. In this step, we focus on 50%, 25%, 10% and 1% only.

Mathematical Talk

Why do we divide a quantity by 2 in order to find 50%?

How do you calculate 10% of a number mentally?

What's the same and what's different about 10% of 300 and 10% of 30?

Varied Fluency

Eva says,

- 50% is equivalent to \(\frac{1}{2}\)
- To find 50% of an amount, I can divide by 2

Complete the sentences.

25% is equivalent to \(\frac{1}{4}\) To find 25% of an amount, divide by __

10% is equivalent to \(\frac{1}{10}\) To find 10% of an amount, divide by __

1% is equivalent to \(\frac{1}{100}\) To find 1% of an amount, divide by __

Use the bar models to help you complete the calculations.

Find:

- 50% of 406 =
- 25% of 124 =
- 50% of 300
- 25% of 300
- 10% of 300
- 1% of 300
- 50% of 30
- 25% of 30
- 10% of 30
- 1% of 30
- 50% of 60
- 25% of 60
- 10% of 60
- 1% of 60
Reasoning and Problem Solving

Mo says,

To find 10% you divide by 10, so to find 50% you divide by 50

Do you agree? Explain why.

Possible answer:

Mo is wrong because 50% is equivalent to a half so to find 50% you divide by 2

Eva says to find 1% of a number, you divide by 100
Whitney says to find 1% of a number, you divide by 10 and then by 10 again.

Who do you agree with? Explain your answer.

They are both correct. Whitney has divided by 100 in two smaller steps.

Complete the missing numbers.

50% of 40 = ___% of 80
___% of 40 = 1% of 400
10% of 500 = ___% of 100

25
10
50
Varied Fluency

Mo uses a bar model to find 30% of 220

\[
\text{10\% of 220} = 22, \text{ so } 30\% \text{ of 220} = 3 \times 22 = 66
\]

Use Mo’s method to calculate:

- 40% of 220
- 20% of 110
- 30% of 440
- 90% of 460

Mathematical Talk

Is dividing by 10 and multiplying by 5 the most efficient way to find 50%? Explain why.

Is dividing by 10 and multiplying by 9 the most efficient way to find 90%? Explain why.

How many ways can you think of to calculate 60% of a number?

How else could we work out 5%?

Calculate:

- 15% of 60 m
- 35% of 300 g
- 65% of £20

Children build on the last step by finding multiples of 10% and other known percentages.
They explore different methods of finding certain percentages e.g. Finding 20% by dividing by 10 and multiplying by 2 or by dividing by 5. They also explore finding 5% by finding half of 10%. Using these methods, children build up to find percentages such as 35%.
Reasoning and Problem Solving

Four children in a class were asked to find 20% of an amount, this is what they did:

- Whitney: I divided by 5 because 20% is the same as one fifth.
- Amir: I found one percent by dividing by 100, then I multiplied my answer by 20.
- Alex: I did 10% add 10%.
- Jack: I found ten percent by dividing by 10, then I multiplied my answer by 2.

Who do you think has the most efficient method? Explain why. Who do you think will end up getting the answer incorrect?

All methods are acceptable ways of finding 20%. Children may have different answers because they may find different methods easier. Discussion could be had around whether or not their preferred method is always the most efficient.

Possible methods include:
- $10\% \times 4 + 5\%$
- $25\% + 20\%$
- $25\% + 10\% + 10\%$
- $50\% - 5\%$
- To find 60% of 45: $10\% \times 6$
- $50\% + 10\%$
- $10\% \times 3$

Children will notice that 45% of 60 = 60% of 45. This always happens.
350,000 people visited the Natural History Museum last week. 
15% of the people visited on Monday. 
40% of the people visited on Saturday. 
How many people visited the Natural History Museum during the rest of the week?

If 7 is 10% of a number, what is the number? 

Use the bar model to help you.

Complete:

10% of 150 = [ ]
30% of [ ] = 45
30% of 300 = [ ]
30% of [ ] = 900

Can you see a link between the questions?
### Percentages – Missing Values

**Reasoning and Problem Solving**

<table>
<thead>
<tr>
<th>What percentage questions can you ask about this bar model?</th>
<th>Possible answer: If 20% of a number is 3.5, what is the whole? What is 60%? What is 10%?</th>
</tr>
</thead>
<tbody>
<tr>
<td>3.5</td>
<td>25% of 60 = 25% of 60</td>
</tr>
</tbody>
</table>

<table>
<thead>
<tr>
<th>Fill in the missing values to make this statement correct. Can you find more than one way?</th>
<th>Possible answers: 25% of 60 = 25% of 60</th>
</tr>
</thead>
<tbody>
<tr>
<td>25% of □ = □ % of 60</td>
<td>25% of 120 = 50% of 60</td>
</tr>
<tr>
<td>25% of 24 = 10% of 60</td>
<td>25% of 2.4 = 1% of 60</td>
</tr>
<tr>
<td>25% of 180 = 75% of 60</td>
<td></td>
</tr>
</tbody>
</table>

**A golf club has 200 members.**

- 58% of the members are male.
- 50% of the female members are children.

(a) How many male members are in the golf club?

(b) How many female children are in the golf club?

- 116 male members
- 42 female children