New for 2020/21

2020 will go down in history. The world has changed for all of us.

We want to do as much as we can to support children, teachers, parents and carers in these very uncertain times.

We have amended our schemes for 2020/21 to:

★ highlight key teaching points

★ recap essential content that children may have forgotten

★ flag any content that you might not have covered during the school closures period.

We hope these changes will add further value to the schemes and save you time.

Lesson-by-lesson overviews

We’ve always been reluctant to produce lesson-by-lesson overviews as every class is individual and has different needs. However, many of you have said that if blended learning becomes a key feature of school life next year, a weekly plan with linked content and videos could be really useful.

As always, we’ve listened! We’ve now produced a complete lesson-by-lesson overview for Y1 to Y9 that schools can use or adapt as they choose. Each lesson will be linked to a free-to-use home learning video, and for premium subscribers, a worksheet. This means that you can easily assign work to your class, whether they are working at home or in school.

Inevitably, this lesson-by-lesson structure won’t suit everyone, but if it works for you, then please do make use of this resource as much as you wish.
Teaching for Mastery

These overviews are designed to support a mastery approach to teaching and learning and have been designed to support the aims and objectives of the new National Curriculum.

The overviews:

- have number at their heart. A large proportion of time is spent reinforcing number to build competency
- ensure teachers stay in the required key stage and support the ideal of depth before breadth.
- ensure students have the opportunity to stay together as they work through the schemes as a whole group
- provide plenty of opportunities to build reasoning and problem solving elements into the curriculum.

For more guidance on teaching for mastery, visit the NCETM website:

https://www.ncetm.org.uk/resources/47230

Concrete - Pictorial - Abstract

We believe that all children, when introduced to a new concept, should have the opportunity to build competency by taking this approach.

**Concrete** – children should have the opportunity to use concrete objects and manipulatives to help them understand what they are doing.

**Pictorial** – alongside this children should use pictorial representations. These representations can then be used to help reason and solve problems.

**Abstract** – both concrete and pictorial representations should support children’s understanding of abstract methods.

Need some CPD to develop this approach? Visit www.whiterosemaths.com for find a course right for you.
Supporting resources

We have produced supporting resources for every small step from Year 1 to Year 11.

The worksheets are provided in three different formats:

- **Write on worksheet** – ideal for children to use the ready made models, images and stem sentences.
- **Display version** – great for schools who want to cut down on photocopying.
- **PowerPoint version** – one question per slide. Perfect for whole class teaching or mixing questions to make your own bespoke lesson.

For more information visit our online training and resources centre [resources.whiterosemaths.com](http://resources.whiterosemaths.com) or email us directly at [support@whiterosemaths.com](mailto:support@whiterosemaths.com)
Meet the Characters

Children love to learn with characters and our team within the scheme will be sure to get them talking and reasoning about mathematical concepts and ideas. Who’s your favourite?
<table>
<thead>
<tr>
<th>Week 1</th>
<th>Week 2</th>
<th>Week 3</th>
<th>Week 4</th>
<th>Week 5</th>
<th>Week 6</th>
<th>Week 7</th>
<th>Week 8</th>
<th>Week 9</th>
<th>Week 10</th>
<th>Week 11</th>
<th>Week 12</th>
</tr>
</thead>
<tbody>
<tr>
<td>Autumn</td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td></td>
<td>Number: Place Value</td>
<td></td>
<td>Number: Addition and Subtraction</td>
<td></td>
<td>Measurement: Length and Perimeter</td>
<td></td>
<td>Number: Multiplication and Division</td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>Spring</td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td></td>
<td>Number: Multiplication and Division</td>
<td></td>
<td>Measurement: Area</td>
<td></td>
<td>Number: Fractions</td>
<td></td>
<td>Number: Decimals</td>
<td></td>
<td></td>
<td></td>
<td>Consolidation</td>
</tr>
<tr>
<td>Summer</td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
</tbody>
</table>
# Overview

## Small Steps

- 11 and 12 times-table
- Multiply 3 numbers
- Factor pairs
- Efficient multiplication
- Written methods
- Multiply 2-digits by 1-digit (1)
- Multiply 2-digits by 1-digit
- Multiply 3-digits by 1-digit
- Divide 2-digits by 1-digit (1)
- Divide 2-digits by 1-digit (1)

---

## Notes for 2020/21

These steps may look similar but these are difficult concepts and children need to spend time exploring different representations of multiplication with no exchange before moving on. They need to use manipulatives to support understanding and make links with repeated addition.

Similarly with division, children will first need to explore examples with no exchange or remainders, making links to the inverse.
Overview
Small Steps

- Divide 2-digits by 1-digit (2)
- Divide 2-digits by 1-digit (2)
- Divide 3-digits by 1-digit
- Correspondence problems

Notes for 2020/21

The final division steps introduce remainders and begin to look at generalisations.
Continue to use place value counters and visual models to support understanding.
11 and 12 Times-table

Notes and Guidance

Building on their knowledge of the 1, 2 and 10 times-tables, children explore the 11 and 12 times-tables through partitioning. They use Base 10 equipment to build representations of the times-tables and use them to explore the inverse of multiplication and division statements. Highlight the importance of commutativity as children should already know the majority of facts from other times-tables.

Mathematical Talk

Which multiplication and division facts in the 11 and 12 times-tables have not appeared before in other times-tables?

Can you partition 11 and 12 into tens and ones? What times-tables can we add together to help us multiply by 11 and 12?

If I know $11 \times 10$ is equal to 110, how can I use this to calculate $11 \times 11$?

Varied Fluency

Fill in the blanks.

<table>
<thead>
<tr>
<th>2 × 10 = ___</th>
<th>2 x 1 = ___</th>
</tr>
</thead>
<tbody>
<tr>
<td>2 lots of 10 doughnuts = ___</td>
<td>2 lots of 1 doughnut = ___</td>
</tr>
<tr>
<td>2 x 10 + 2 x 1 = 2 x 11 = ___</td>
<td></td>
</tr>
</tbody>
</table>

Use Base 10 to build the 12 times-table. e.g.

| 3 x 12 = |

Complete the calculations.

<table>
<thead>
<tr>
<th>12 × 5 = □</th>
<th>5 × 12 = □</th>
<th>48 ÷ 12 = □</th>
<th>84 ÷ 12 = □</th>
</tr>
</thead>
<tbody>
<tr>
<td>12 × □ = 120</td>
<td>12 × □ = 132</td>
<td>□ ÷ 12 = 8</td>
<td>□ = 9 × 12</td>
</tr>
</tbody>
</table>

There are 11 players on a football team. 7 teams take part in a tournament. How many players are there altogether in the tournament?
Here is one batch of muffins.

Teddy bakes 11 batches of muffins. How many muffins does he have altogether?

In each batch there are 3 strawberry, 3 vanilla, 4 chocolate and 2 toffee muffins. How many of each type of muffin does Teddy have in 11 batches?

Teddy sells 5 batches of muffins. How many muffins does he have left?

Teddy has 132 muffins altogether.

Strawberry: 33
Vanilla: 33
Chocolate: 44
Toffee: 22

132 – 55 = 77

Teddy has 77 muffins left.

Rosie uses a bar model to represent 88 divided by 11

Explain Rosie’s mistake.

Can you draw a bar model to represent 88 divided by 11 correctly?

Rosie has divided by grouping in 11s but has found 11 groups of 11 which is equal to 121.

To divide 88 by sharing into 11 equal groups, there would be 8 in each group.

To divide 88 by grouping in 11s, there would be 8 groups of 11.
Children are introduced to the ‘Associative Law’ to multiply 3 numbers. This law focuses on the idea that it doesn’t matter how we group the numbers when we multiply e.g. $4 \times 5 \times 2 = (4 \times 5) \times 2 = 20 \times 2 = 40$

or $4 \times 5 \times 2 = 4 \times (5 \times 2) = 4 \times 10 = 40$

They link this idea to commutativity and see that we can change the order of the numbers to group them more efficiently, e.g. $4 \times 2 \times 5 = (4 \times 2) \times 5 = 8 \times 5 = 40$

Can you use concrete materials to build the calculations?

How will you decide which order to do the multiplication in?

What’s the same and what’s different about the arrays?

Which order do you find easier to calculate efficiently?

Complete the calculations.

Use counters or cubes to represent the calculations.

Choose which order you will complete the multiplication.

- $5 \times 2 \times 6$
- $8 \times 4 \times 5$
- $2 \times 8 \times 6$
Reasoning and Problem Solving

Choose three digit cards. Arrange them in the calculation.

\[
\square \times \square \times \square = \square
\]

How many different calculations can you make using your three digit cards? Which order do you find it the most efficient to calculate the product? How have you grouped the numbers?

Possible answers using 3, 4 and 7:

\[
\begin{align*}
7 \times 3 \times 4 &= 84 \\
7 \times 4 \times 3 &= 84 \\
4 \times 3 \times 7 &= 84 \\
4 \times 7 \times 3 &= 84 \\
3 \times 4 \times 7 &= 84 \\
3 \times 7 \times 4 &= 84
\end{align*}
\]

Children may find it easier to calculate \(7 \times 3\) first and then multiply it by 4 as 21 multiplied by 4 has no exchanges.

Make the target number of 84 using three of the digits below.

\[
\begin{array}{cccc}
7 & 5 & 3 & 4 \\
\end{array}
\]

\[
\square \times \square \times \square = 84
\]

Multiply the remaining three digits together, what is the product of the three numbers?

Is the product smaller or larger than 84?

Can you complete this problem in more than one way?

Possible answers:

\[
\begin{align*}
7 \times 2 \times 6 &= 84 \\
4 \times 3 \times 5 &= 60 \\
60 \text{ is smaller than } 84 \\
7 \times 3 \times 4 &= 84 \\
2 \times 6 \times 5 &= 60 \\
60 \text{ is smaller than } 84 \\
\end{align*}
\]

Children may also show the numbers in a different order.
Children learn that a factor is a whole number that multiplies by another number to make a product e.g. $3 \times 5 = 15$, factor $\times$ factor $=$ product.

They develop their understanding of factor pairs using concrete resources to work systematically, e.g. factor pairs for 12 – begin with $1 \times 12$, $2 \times 6$, $3 \times 4$. At this stage, children recognise that they have already used 4 in the previous calculation therefore all factor pairs have been identified.

12 has ____ factor pairs. 12 has ___ factors altogether.

Use counters to create arrays for 24

How many factor pairs can you find?

Which number is a factor of every whole number?

Do factors always come in pairs?

Do whole numbers always have an even number of factors?

How do arrays support in finding factors of a number?

How do arrays support us in seeing when a number is not a factor of another number?

Here is an example of a factor bug for 12

Complete the factor bug for 36

Are all the factors in pairs?

Draw your own factor bugs for 16, 48, 56 and 35
Reasoning and Problem Solving

Tommy says

The greater the number, the more factors it will have.

Is Tommy correct?

Use arrays to explain your answer.

Tommy is incorrect.
Children explain by showing an example of two numbers where the greater number has less factors.
For example, 15 has 4 factors 1, 3, 5 and 15
17 has 2 factors 1 and 17

Some numbers are equal to the sum of all their factors (not including the number itself).
e.g. 6
6 has 4 factors, 1, 2, 3 and 6
Add up all the factors not including 6 itself.
1 + 2 + 3 = 6
6 is equal to the sum of its factors (not including the number itself)

How many other numbers can you find that are equal to the sum of their factors?
Which numbers are less than the sum of their factors?
Which numbers are greater than the sum of their factors?

Possible answers
28 = 1 + 2 + 4 + 7 + 14
28 is equal to the sum of its factors.
12 < 1 + 2 + 3 + 4 + 6
12 is less than the sum of its factors.
8 > 1 + 2 + 4
8 is greater than the sum of its factors.
Children develop their mental multiplication by exploring different ways to calculate. They partition two-digit numbers into tens and ones or into factor pairs in order to multiply one and two-digit numbers. By sharing mental methods, children can learn to be more flexible and efficient.

Class 4 are calculating 25 \times 8 mentally. Can you complete the calculations in each of the methods?

**Method 1**
\[
25 \times 8 = 20 \times 8 + 5 \times 8
= 160 + 40 = 200
\]

**Method 2**
\[
25 \times 8 = 5 \times 5 \times 8
= 5 \times 40 = 200
\]

**Method 3**
\[
25 \times 8 = 25 \times 10 - 25 \times 2
= 250 - 50 = 200
\]

**Method 4**
\[
25 \times 8 = 50 \times 8 \div 2
= 50 \div 2 = 25
\]

Can you think of any other ways to mentally calculate 25 \times 8? Which do you think is the most efficient? How would you calculate 228 \times 5 mentally?
Reasoning and Problem Solving

Teddy has calculated $19 \times 3$

20 $\times$ 3 = 60
60 $-$ 1 = 59
19 $\times$ 3 = 59

Can you explain his mistake and correct the diagram?

Here are three number cards.

| 21 | 42 | 38 |

Dora, Annie and Eva choose one of the number cards each. They multiply their number by 5.

Dora says,

I did 40 $\times$ 5 and then subtracted 2 lots of five.

Annie says,

I multiplied my number by 10 and then divided 210 by 2.

Eva says,

I halved my 2-digit number and doubled 5 so I calculated 21 $\times$ 10

Which number card did each child have? Would you have used a different method to multiply the numbers by 5?

Dora has 38
Annie has 21
Eva has 42

Children can then discuss the methods they would have used and why.
Children use a variety of informal written methods to multiply a two-digit and a one-digit number. It is important to emphasise when it would be more efficient to use a mental method to multiply and when we need to represent our thinking by showing working.

There are 8 classes in a school. Each class has 26 children. How many children are there altogether? Complete the number line to solve the problem.

Use this method to work out the multiplications:
- \(16 \times 7\)
- \(34 \times 6\)
- \(27 \times 4\)

Rosie uses Base 10 and a part-whole model to calculate \(26 \times 3\). Complete Rosie’s calculations.

Use Rosie’s method to work out:
- \(36 \times 3\)
- \(24 \times 6\)
- \(45 \times 4\)

Why are there not 26 jumps of 8 on the number line?

Could you find a more efficient method?

Can you calculate the multiplication mentally or do you need to write down your method?

Can you partition your number into more than two parts?
Here are 6 multiplications.

<table>
<thead>
<tr>
<th>43 \times 5</th>
<th>54 \times 6</th>
<th>38 \times 6</th>
</tr>
</thead>
<tbody>
<tr>
<td>33 \times 2</td>
<td>19 \times 7</td>
<td>84 \times 5</td>
</tr>
</tbody>
</table>

Which of the multiplications would you calculate mentally?
Which of the multiplications would you use a written method for?

Explain your choices to a partner. Did your partner choose the same methods as you?

Children will sort the multiplications in different ways.

It is important that teachers discuss with the children why they have made the choices and refer back to the efficient multiplication step to remind children of efficient ways to multiply mentally.

Ron is calculating 46 multiplied by 4 using the part-whole model.

\[
\begin{align*}
46 \times 4 &= 1,624 \\
40 \times 4 &= 160 \\
6 \times 4 &= 24
\end{align*}
\]

Can you explain Ron's mistake?

Ron has multiplied the parts correctly, but added them up incorrectly.

\[
160 + 24 = 184
\]
Children use their understanding of repeated addition to represent a two-digit number multiplied by a one-digit number with concrete manipulatives. They use the formal method of column multiplication alongside the concrete representation. They also apply their understanding of partitioning to represent and solve calculations.

In this step, children explore multiplication with no exchange.

### Mathematical Talk

How does multiplication link to addition?

How does partitioning help you to multiply 2-digits by a 1-digit number?

How does the written method match the concrete representation?

### Varied Fluency

There are 21 coloured balls on a snooker table. How many coloured balls are there on 3 snooker tables?

Use Base 10 to calculate: 21 × 4 and 33 × 3

Complete the calculations to match the place value counters.

Annie uses place value counters to work out 34 × 2

Use Annie’s method to solve:

- 23 × 3
- 32 × 3
- 42 × 2
Reasoning and Problem Solving

Alex completes the calculation:

43 × 2

Can you spot her mistake?

<table>
<thead>
<tr>
<th>T</th>
<th>O</th>
</tr>
</thead>
<tbody>
<tr>
<td>4</td>
<td>3</td>
</tr>
<tr>
<td>×</td>
<td>2</td>
</tr>
<tr>
<td></td>
<td>6</td>
</tr>
<tr>
<td>+</td>
<td>8</td>
</tr>
<tr>
<td></td>
<td>1</td>
</tr>
</tbody>
</table>

Alex has multiplied 4 by 2 rather than 40 by 2.

Teddy completes the same calculation as Alex.
Can you spot and explain his mistake?

<table>
<thead>
<tr>
<th></th>
<th>T</th>
<th>O</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>4</td>
<td>3</td>
</tr>
<tr>
<td>×</td>
<td>2</td>
<td></td>
</tr>
<tr>
<td></td>
<td>8</td>
<td>0</td>
</tr>
<tr>
<td></td>
<td>6</td>
<td></td>
</tr>
</tbody>
</table>

Teddy has written 80 where he should have just put an 8 because he is multiplying 4 tens by 2 which is 8 tens. The answer should be 86.

Dexter says,

4 × 21 = 2 × 42

Is Dexter correct?

True. Both multiplications are equal to 84.

Children may explore that one number has halved and the other has doubled.
Children build on their understanding of formal multiplication from Year 3 to move to the formal short multiplication method. Children use their knowledge of exchanging ten ones for one ten in addition and apply this to multiplication, including exchanging multiple groups of tens. They use place value counters to support their understanding.

### Notes and Guidance

Which column should we start with, the ones or the tens?

How are Ron and Whitney’s methods the same?

How are they different?

Can we write a list of key things to remember when multiplying using the column method?

### Varied Fluency

#### Whitney uses place value counters to calculate $5 \times 34$

![Place Value Counter Diagram](image)

Use Whitney’s method to solve:

- $5 \times 42$
- $23 \times 6$
- $48 \times 3$

#### Ron also uses place value counters to calculate $5 \times 34$

![Place Value Counter Diagram](image)

Use Ron’s method to complete:

- $4 \times 3$
- $3 \times 4$
- $7 \times 5$
Multiply 2-digits by 1-digit

Reasoning and Problem Solving

Here are three incorrect multiplications.

<table>
<thead>
<tr>
<th>T</th>
<th>O</th>
</tr>
</thead>
<tbody>
<tr>
<td>6</td>
<td>1</td>
</tr>
<tr>
<td>x</td>
<td>5</td>
</tr>
<tr>
<td>---</td>
<td>---</td>
</tr>
<tr>
<td>3</td>
<td>5</td>
</tr>
</tbody>
</table>

<table>
<thead>
<tr>
<th>T</th>
<th>O</th>
</tr>
</thead>
<tbody>
<tr>
<td>7</td>
<td>4</td>
</tr>
<tr>
<td>x</td>
<td>7</td>
</tr>
<tr>
<td>---</td>
<td>---</td>
</tr>
<tr>
<td>4</td>
<td>9</td>
</tr>
</tbody>
</table>

<table>
<thead>
<tr>
<th>T</th>
<th>O</th>
</tr>
</thead>
<tbody>
<tr>
<td>2</td>
<td>6</td>
</tr>
<tr>
<td>x</td>
<td>4</td>
</tr>
<tr>
<td>---</td>
<td>---</td>
</tr>
<tr>
<td>8</td>
<td>2</td>
</tr>
</tbody>
</table>

Correct the multiplications.

<table>
<thead>
<tr>
<th>T</th>
<th>O</th>
</tr>
</thead>
<tbody>
<tr>
<td>6</td>
<td>1</td>
</tr>
<tr>
<td>x</td>
<td>5</td>
</tr>
<tr>
<td>---</td>
<td>---</td>
</tr>
<tr>
<td>3</td>
<td>0</td>
</tr>
</tbody>
</table>

<table>
<thead>
<tr>
<th>T</th>
<th>O</th>
</tr>
</thead>
<tbody>
<tr>
<td>7</td>
<td>4</td>
</tr>
<tr>
<td>x</td>
<td>7</td>
</tr>
<tr>
<td>---</td>
<td>---</td>
</tr>
<tr>
<td>5</td>
<td>1</td>
</tr>
</tbody>
</table>

<table>
<thead>
<tr>
<th>T</th>
<th>O</th>
</tr>
</thead>
<tbody>
<tr>
<td>2</td>
<td>6</td>
</tr>
<tr>
<td>x</td>
<td>4</td>
</tr>
<tr>
<td>---</td>
<td>---</td>
</tr>
<tr>
<td>1</td>
<td>0</td>
</tr>
</tbody>
</table>

Always, sometimes, never

- When multiplying a two-digit number by a one-digit number, the product has 3 digits.
- When multiplying a two-digit number by 8 the product is odd.
- When multiplying a two-digit number by 7 you need to exchange.

Prove it.

Sometimes: 12 \times 2 has only two-digits; 23 \times 5 has three digits.

Never: all multiples of 8 are even.

Sometimes: most two-digit numbers need exchanging, but not 10 or 11.
Multiply 3-digits by 1-digit

Notes and Guidance

Children build on previous steps to represent a three-digit number multiplied by a one-digit number with concrete manipulatives. Teachers should be aware of misconceptions arising from 0 in the tens or ones column. Children continue to exchange groups of ten ones for tens and record this in a written method.

Mathematical Talk

How is multiplying a three-digit number by one-digit similar to multiplying a two-digit number by one-digit?

Would you use counters to represent 84 multiplied by 8? Why?

Varied Fluency

Complete the calculation.

A school has 4 house teams. There are 245 children in each house team. How many children are there altogether?

Write the multiplication represented by the counters and calculate the answer using the formal written method.
Spot the mistake

Alex and Dexter have both completed the same multiplication.

Alex

<table>
<thead>
<tr>
<th>H</th>
<th>T</th>
<th>O</th>
</tr>
</thead>
<tbody>
<tr>
<td>2</td>
<td>3</td>
<td>4</td>
</tr>
<tr>
<td>×</td>
<td></td>
<td>6</td>
</tr>
<tr>
<td>1</td>
<td>2</td>
<td>0</td>
</tr>
</tbody>
</table>

Dexter

<table>
<thead>
<tr>
<th>H</th>
<th>T</th>
<th>O</th>
</tr>
</thead>
<tbody>
<tr>
<td>2</td>
<td>3</td>
<td>4</td>
</tr>
<tr>
<td>×</td>
<td></td>
<td>6</td>
</tr>
<tr>
<td>1</td>
<td>4</td>
<td>0</td>
</tr>
</tbody>
</table>

Who has the correct answer? What mistake has been made by one of the children?

Dexter has the correct answer. Alex has forgotten to add the two hundreds she exchanged from the tens column.

Teddy and his mum were having a reading competition. In one month, Teddy read 814 pages.

His mum read 4 times as many pages as Teddy.

How many pages did they read altogether? How many fewer pages did Teddy read?

Use the bar model to help.

Teddy: 814
Mum: 4 × 814 = 3256

814 × 5 = 4,070
They read 4,070 pages altogether.

814 × 3 = 2,442
Teddy read 2,442 fewer pages than his mum.
Notes and Guidance

Children divide 2-digit numbers by a 1-digit number by partitioning into tens and ones and sharing into equal groups.

They divide numbers that involve exchanging between the tens and ones. The answers do not have remainders.

Children use their times-tables to partition the number into multiples of the divisor.

Mathematical Talk

Why have we partitioned 42 into 30 and 12 instead of 40 and 2?

What do you notice about the partitioned numbers and the divisor?

Why do we partition 96 in different ways depending on the divisor?

Varied Fluency

Ron uses place value counters to divide 42 into three equal groups.

He shares the tens first and exchanges the remaining ten for ones.

Then he shares the ones. $42 ÷ 3 = 14$

Use Ron’s method to calculate $48 ÷ 3$, $52 ÷ 4$ and $92 ÷ 8$

Annie uses a similar method to divide 42 by 3

Use Annie’s method to calculate:

$96 ÷ 8$, $96 ÷ 4$, $96 ÷ 3$, $96 ÷ 6$
Amir partitioned a number to help him divide by 8. Some of his working out has been covered with paint. What number could Amir have started with? The answer could be 56 or 96.

Compare the statements using <, > or =

- $48 \div 4$  $\bigcirc$  $36 \div 3$ =
- $52 \div 4$  $\bigcirc$  $42 \div 3$ <
- $60 \div 3$  $\bigcirc$  $60 \div 4$ >
Notes and Guidance

Children build on their knowledge of dividing a 2-digit number by a 1-digit number from Year 3 by sharing into equal groups.

Children use examples where the tens and the ones are divisible by the divisor, e.g. 96 divided by 3 and 84 divided by 4. They then move on to calculations where they exchange between tens and ones.

Mathematical Talk

How can we partition 84?  
How many rows do we need to share equally between?  
If I cannot share the tens equally, what do I need to do?  
How many ones will I have after exchanging the tens?

If we know $96 \div 4 = 24$, what will $96 \div 8$ be?  
What will $96 \div 2$ be? Can you spot a pattern?

Varied Fluency

Jack is dividing 84 by 4 using place value counters.

First, he divides the tens.  
Then, he divides the ones.

Use Jack’s method to calculate:

$69 \div 3$  
$88 \div 4$  
$96 \div 3$

Rosie is calculating 96 divided by 4 using place value counters.  
First, she divides the tens. She has one ten remaining so she exchanges one ten for ten ones. Then, she divides the ones.

Use Rosie’s method to solve:

$65 \div 5$  
$75 \div 5$  
$84 \div 6$
### Divide 2-digits by 1-digit (1)

#### Reasoning and Problem Solving

<table>
<thead>
<tr>
<th>Dora is calculating $72 \div 3$ Before she starts, she says the calculation will involve an exchange.</th>
<th>Dora is correct because 70 is not a multiple of 3 so when you divide 7 tens between 3 groups there will be one remaining which will be exchanged.</th>
<th>Eva has 96 sweets. She shares them into equal groups. She has no sweets left over. How many groups could Eva have shared her sweets into?</th>
</tr>
</thead>
<tbody>
<tr>
<td>Do you agree? Explain why.</td>
<td></td>
<td>Possible answers</td>
</tr>
</tbody>
</table>

<table>
<thead>
<tr>
<th>Use $&lt;$, $&gt;$ or $=$ to complete the statements.</th>
<th></th>
<th></th>
</tr>
</thead>
<tbody>
<tr>
<td>$69 \div 3$  $96 \div 3$</td>
<td>$&lt; $</td>
<td>$96 \div 1 = 96$</td>
</tr>
<tr>
<td>$96 \div 4$  $96 \div 3$</td>
<td>$&lt; $</td>
<td>$96 \div 2 = 48$</td>
</tr>
<tr>
<td>$91 \div 7$  $84 \div 6$</td>
<td>$&lt; $</td>
<td>$96 \div 3 = 32$</td>
</tr>
<tr>
<td></td>
<td></td>
<td>$96 \div 4 = 24$</td>
</tr>
<tr>
<td></td>
<td></td>
<td>$96 \div 6 = 16$</td>
</tr>
<tr>
<td></td>
<td></td>
<td>$96 \div 8 = 12$</td>
</tr>
</tbody>
</table>
Divide 2-digits by 1-digit (3)

Notes and Guidance

Children move onto solving division problems with a remainder.
Links are made between division and repeated subtraction, which builds on learning in Year 2.
Children record the remainders as shown in Tommy’s method.
This notation is new to Year 3 so will need a clear explanation.

Mathematical Talk

How do we know 13 divided by 4 will have a remainder?

Can a remainder ever be more than the divisor?

Which is your favourite method?
Which methods are most efficient with larger two digit numbers?

Varied Fluency

How many squares can you make with 13 lollipop sticks?
There are ___ lollipop sticks.
There are ___ groups of 4
There is ___ lollipop stick remaining.
13 ÷ 4 = ___ remainder ___
Use this method to see how many triangles you can make with 38 lollipop sticks.

Tommy uses repeated subtraction to solve 31 ÷ 4

Use Tommy’s method to solve 38 divided by 3

Use place value counters to work out 94 ÷ 4
Did you need to exchange any tens for ones?
Is there a remainder?
## Divide 2-digits by 1-digit (3)

### Reasoning and Problem Solving

<table>
<thead>
<tr>
<th>Calculation</th>
<th>Explanation</th>
</tr>
</thead>
<tbody>
<tr>
<td>64 ÷ 8</td>
<td>64 ÷ 8 could be the odd one out as it is the only calculation without a remainder. Make sure other answers are considered such as 65 ÷ 3 because it is the only one being divided by an odd number.</td>
</tr>
<tr>
<td>77 ÷ 4</td>
<td></td>
</tr>
<tr>
<td>49 ÷ 6</td>
<td></td>
</tr>
<tr>
<td>65 ÷ 3</td>
<td></td>
</tr>
</tbody>
</table>

Jack has 15 stickers. He sorts his stickers into equal groups but has some stickers remaining. How many stickers could be in each group and how many stickers would be remaining?

Dora and Eva are planting bulbs. They have 76 bulbs altogether. Dora plants her bulbs in rows of 8 and has 4 left over. Eva plants her bulbs in rows of 10 and has 2 left over. How many bulbs do they each have?

Dora has 44 bulbs. Eva has 32 bulbs.
Divide 2-digits by 1-digit (2)

Notes and Guidance

Children explore dividing 2-digit numbers by 1-digit numbers involving remainders.

They continue to use the place value counters to divide in order to explore why there are remainders. Teachers should highlight, through questioning, that the remainder can never be greater than the number you are dividing by.

Mathematical Talk

If we are dividing by 3, what is the highest remainder we can have?

If we are dividing by 4, what is the highest remainder we can have?

Can we make a general rule comparing our divisor (the number we are dividing by) to our remainder?

Varied Fluency

Teddy is dividing 85 by 4 using place value counters.

First, he divides the tens.

Then, he divides the ones.

Use Teddy's method to calculate:

86 ÷ 4  87 ÷ 4  88 ÷ 4  97 ÷ 3  98 ÷ 3  99 ÷ 3

Whitney uses the same method, but some of her calculations involve an exchange.

Use Whitney's method to solve:

57 ÷ 4  58 ÷ 4  58 ÷ 3
### Divide 2-digits by 1-digit (2)

#### Reasoning and Problem Solving

| Rosie writes,  
85 ÷ 3 = 28 r 1  
She says 85 must be 1 away from a multiple of 3  
Do you agree? | I agree, remainder 1 means there is 1 left over. 85 is one more than 84 which is a multiple of 3 | Whitney is thinking of a 2-digit number that is less than 50  
When it is divided by 2, there is no remainder.  
When it is divided by 3, there is a remainder of 1  
When it is divided by 5, there is a remainder of 3 | Whitney is thinking of 28 |
| --- | --- | --- | --- |
| 37 sweets are shared between 4 friends. How many sweets are left over?  
Four children attempt to solve this problem.  
- Alex says it’s 1  
- Mo says it’s 9  
- Eva says it’s 9 r 1  
- Jack says it’s 8 r 5  
Can you explain who is correct and the mistakes other people have made? | Alex is correct as there will be one remaining sweet. Mo has found how many sweets each friend will receive. Eva has written the answer to the calculation. Jack has found a remainder that is larger than the divisor so is incorrect. | Alex is correct as there will be one remaining sweet. Mo has found how many sweets each friend will receive. Eva has written the answer to the calculation. Jack has found a remainder that is larger than the divisor so is incorrect. | |
Divide 3-digits by 1-digit

Notes and Guidance

Children apply their previous knowledge of dividing 2-digit numbers to divide a 3-digit number by a 1-digit number.

They use place value counters and part-whole models to support their understanding.

Children divide numbers with and without remainders.

Mathematical Talk

What is the same and what’s different when we are dividing 3-digit number by a 1-digit number and a 2-digit number by a 1-digit number?

Do we need to partition 609 into three parts or could it just be partitioned into two parts?

Can we partition the number in more than one way to support dividing more efficiently?

Varied Fluency

Annie is dividing 609 by 3 using place value counters.

<table>
<thead>
<tr>
<th>Hundreds</th>
<th>Tens</th>
<th>Ones</th>
</tr>
</thead>
<tbody>
<tr>
<td>100</td>
<td>100</td>
<td>11</td>
</tr>
<tr>
<td>100</td>
<td>100</td>
<td>11</td>
</tr>
<tr>
<td>100</td>
<td>100</td>
<td>11</td>
</tr>
</tbody>
</table>

Use Annie’s method to calculate the divisions.

906 ÷ 3  884 ÷ 4  884 ÷ 8  489 ÷ 2

Rosie is using flexible partitioning to divide 3-digit numbers. She uses her place value counters to support her.

Use Rosie’s method to solve:

726 ÷ 6
846 ÷ 6
846 ÷ 7
Reasoning and Problem Solving

Divide 3-digits by 1-digit

Dexter is calculating $208 \div 8$ using part-whole models.
Can you complete each model?

$208 \div 8 = 26$
$80 \div 8 = 10$
$48 \div 8 = 6$
$160 \div 8 = 20$
$40 \div 8 = 5$
$8 \div 8 = 1$

Children can then make a range of part-whole models to calculate $132 \div 4$

$208 \div 8 = 26$
$80 \div 8 = 10$
$48 \div 8 = 6$
$160 \div 8 = 20$
$40 \div 8 = 5$
$8 \div 8 = 1$

Create a 3-digit number divisible by 2
Create a 3-digit number divisible by 3
Create a 3-digit number divisible by 4
Create a 3-digit number divisible by 5
Can you find a 3-digit number divisible by 6, 7, 8 or 9?

You have 12 counters and the place value grid. You must use all 12 counters to complete the following.

- 2: Any even number
- 3: Any 3-digit number (as the digits add up to 12, a multiple of 3)
- 4: A number where the last two digits are a multiple of 4
- 5: Any number with 0 or 5 in the ones column.

Possible answers

- 6: Any even number
- 7: 714, 8: 840
- 9: impossible
Children solve more complex problems building on their understanding from Year 3 of when $n$ objects relate to $m$ objects.

They find all solutions and notice how to use multiplication facts to solve problems.

**Mathematical Talk**

Can you use a table to support you to find all the combinations?

Can you use a code to help you find the combinations? e.g. VS meaning Vanilla and Sauce

Can you use coins to support you to make all the possible combinations?

**An ice-cream van has 4 flavours of ice-cream and 2 choices of toppings.**

<table>
<thead>
<tr>
<th>Ice-cream flavour</th>
<th>Toppings</th>
</tr>
</thead>
<tbody>
<tr>
<td>Vanilla</td>
<td>Sauce</td>
</tr>
<tr>
<td>Chocolate</td>
<td>Flake</td>
</tr>
<tr>
<td>Strawberry</td>
<td></td>
</tr>
<tr>
<td>Banana</td>
<td></td>
</tr>
</tbody>
</table>

How many different combinations of ice-cream and toppings can be made?

Complete the multiplication to represent the combinations.

___ × ___ = ___ There are ___ combinations.

**Jack has two piles of coins.**

He chooses one coin from each pile.

What are all the possible combinations of coins Jack can choose?

What are all the possible totals he can make?
Here are the meal choices in the school canteen.

<table>
<thead>
<tr>
<th>Starter</th>
<th>Main</th>
<th>Dessert</th>
</tr>
</thead>
<tbody>
<tr>
<td>Soup</td>
<td>Pasta</td>
<td>Cake</td>
</tr>
<tr>
<td>Garlic Bread</td>
<td>Chicken</td>
<td>Ice-cream Cake</td>
</tr>
<tr>
<td>Beef</td>
<td>Beef</td>
<td>Fruit Salad</td>
</tr>
<tr>
<td>Salad</td>
<td>Salad</td>
<td></td>
</tr>
</tbody>
</table>

There are 2 choices of starter, 4 choices of main and 3 choices of dessert.

How many meal combinations can you find? Can you use a systematic approach? Can you represent the combinations in a multiplication?

If there were 20 meal combinations, how many starters, mains and desserts might there be?

There are 24 meal combinations altogether.
\[2 \times 4 \times 3 = 24\]

20 combinations
- \[1 \times 1 \times 20\]
- \[1 \times 2 \times 10\]
- \[1 \times 4 \times 5\]
- \[2 \times 2 \times 5\]

Accept all other variations of these four multiplications e.g. \[1 \times 20 \times 1\]

Alex has 6 T-shirts and 4 pairs of shorts.
Dexter has 12 T-shirts and 2 pairs of shorts.
Who has the most combinations of T-shirts and shorts? Explain your answer.

Alex and Dexter have the same number of combinations of T-shirts and shorts.
Reasoning and Problem Solving

Spring - Block 2

Area
Overview

Small Steps

- What is area?
- Counting squares
- Making shapes
- Comparing area

Notes for 2020/21

This is brand new learning for children. Opportunities for exploration of vocabulary is key. Make sure children cover larger surfaces and have a clear understanding of the concept of area before moving onto counting small squares.
Children are introduced to area for the first time. They understand that area is the amount of space taken up by a 2D shape or surface. Children investigate different shapes that can be made with sets of sticky notes. They should be encouraged to see that the same number of sticky notes can make different shapes but they cover the same amount of surface. We call this the area of a shape.

**What is Area?**

**Notes and Guidance**

Children are introduced to area for the first time. They understand that area is the amount of space taken up by a 2D shape or surface. Children investigate different shapes that can be made with sets of sticky notes. They should be encouraged to see that the same number of sticky notes can make different shapes but they cover the same amount of surface. We call this the area of a shape.

**Mathematical Talk**

Use square sticky notes to find areas of different items in the classroom, which items have the largest surface area? Would we want to find the area of the playground using sticky notes? What else could we use? Why are shapes with perpendicular sides more effective to find the area of rectilinear shapes?

**Varied Fluency**

Which of the two shapes covers most surface?

How do you know?

This is a square sticky note.

Estimate how many sticky notes you need to make these shapes?

Now make the shapes using sticky notes. Which ones cover the largest amount of surface? Which ones cover the least amount of surface?
What is Area?

Reasoning and Problem Solving

Teddy and Eva are measuring the area of the same rectangle.

Teddy uses circles to find the area.

Eva uses squares to find the area.

Whose method do you think is more reliable? Explain why.

Possible answer:
Eva’s method is more reliable than Teddy’s because her squares cover the whole surface of the rectangle whereas the circles leave some of the surface uncovered.

Two children have measured the top of their desk. They used different sized squares.

Dora needed fewer squares to cover the space, so her squares must have been the larger ones. If the squares are smaller, you need more of them.

Who used the largest squares? How do you know?

The area of the table top is 6 squares.

The area of the table top is 9 squares.
Once children understand that area is measured in squares, they use the strategy of counting the number of squares in a shape to measure and compare the areas of rectilinear shapes. They explore the most efficient method of counting squares and link this to their understanding of squares and rectangles.

Mathematical Talk

What strategy can you use to ensure you don’t count a square twice?

Which colour covers the largest area of the quilt?
Which colour covers the smallest area of the quilt?

Will Jack’s method work for every rectilinear shape?

Varied Fluency

Complete the sentences for each shape.

The area of the shape is ____ squares.

Here is a patchwork quilt.
It is made from different coloured squares.
Find the area of each colour.

Purple = ___ squares
Green = ___ squares
Yellow = ___ squares
Orange = ___ squares

Jack uses his times-tables to count the squares more efficiently.

There are 4 squares in 1 row.
There are 3 rows altogether.
3 rows of 4 squares = 12 squares

Use Jack’s method to find the area of this rectangle.
Dexter has taken a bite of the chocolate bar. The chocolate bar was a rectangle. Can you work out how many squares of chocolate there were to start with?

There were 20 squares. You know this because two sides of the rectangle are shown.

This rectangle has been ripped.

What is the smallest possible area of the original rectangle?

What is the largest possible area if the length of the rectangle is less than 10 squares?

Smallest area – 15 squares.

Largest area – 30 squares.
Notes and Guidance

Children make rectilinear shapes using a given number of squares.

It is important that children understand that the rectilinear shapes they make need to touch at the sides not just at the corners. They can work systematically to find all the different rectilinear shapes by moving one square at a time.

Ron has 4 squares.
He systematically makes rectilinear shapes.

Use 5 squares to make rectilinear shapes.
Can you work systematically?

Use squared paper to draw 4 different rectilinear shapes with an area of 12 squares.
Compare your shapes to a partner.
Are they the same?
Are they different?

Mo is building a patio made of 20 square slabs.
What could the patio look like?
Mo is using 6 black square slabs in his design.
None of them are touching each other.
Where could they be in the designs you have made?

Mathematical Talk

If you turn Ron’s shapes upside down, do they stay the same or are they different?

Should you overlap the squares when counting area? Explain your answer.

How many different rectilinear shapes can you make with 8 squares? Will the area always be the same? Why?
Here is a rectilinear shape.

Using 7 more squares, can you make a rectangle? Can you find more than one way?

Possible answers include:

Can you make some capital letters on squared paper using less than 20 squares?

Make a word from some and count the total area of the letters. Which letters have a line of symmetry? What is the area of half of each letter?

Most letters can be made. They could be drawn on large squared paper or made with square tiles.
Children compare the area of rectilinear shapes where the same size square has been used.

Children will be able to use < and > with the value of the area to compare shapes.

They will also put shapes in order of size by comparing their areas.

**Mathematical Talk**

How much larger/smaller is the area of the shape?

How can we order the shapes?

Can we draw a shape that would have the same area as ___?

What is different about the number of squares covered by shape A?

**Notes and Guidance**

**Varied Fluency**

Use the words ‘greater than’ and ‘less than’ to compare the rectilinear shapes.

Complete the sentence stems using < and >

Put the shapes in order from largest to smallest area.

Here is a shape. Draw a shape that has a smaller area than this shape but an area greater than 7 squares.

Draw a shape that has an area equal to the first shape, but looks different.
Reasoning and Problem Solving

Look at the shapes. Can you spot the pattern and explain how the area is changing each time?

The area increases by 2 each time.

The next shape will have an area of 9.

The 6th shape will have an area of 13.

The answers are all odd numbers and increase by 2 each time.

Shape C has been deleted.

Area C > Area B
Area C < Area D

Can you draw what shape C could look like?

Shape A is missing too.

- It has the smallest area.
- It is symmetrical.

Can you draw what it could look like?

Shape B has an area of 18 squares.

Shape D has an area of 21 squares.

So Shape C can be any shape that has an area between 18 and 21 squares.

Shape A must have area less than 18 squares, but can be any symmetrical design e.g. a 4 by 4 square.
Overview

Small Steps

- Unit and non-unit fractions
- What is a fraction?
- Tenths
- Count in tenths
- Equivalent fractions (1)
- Equivalent fractions (2)
- Equivalent fractions (1)
- Equivalent fractions (2)
- Fractions greater than 1
- Count in fractions
- Add fractions
- Add 2 or more fractions

Notes for 2020/21

Year 3 fractions work was in the summer term and learning may have been missed. We have therefore added a number of recap steps to ensure children have a thorough understanding of tenths and equivalent fractions before moving into adding and subtracting.

The progression from paper folding and finding two equivalent fractions is explored before moving onto looking at numerical relationships in a more abstract way.
# Year 4 | Spring Term | Week 5 to 8 – Number: Fractions

## Small Steps

<table>
<thead>
<tr>
<th>Small Step</th>
<th>Notes for 2020/21</th>
</tr>
</thead>
<tbody>
<tr>
<td>Subtract fractions</td>
<td>The recap step here suggests children use practical equipment and pictorial representations to subtract fractions with the same denominator within one whole. They can then apply this to subtracting more than one fraction and from whole amounts.</td>
</tr>
<tr>
<td>Subtract 2 fractions</td>
<td></td>
</tr>
<tr>
<td>Subtract from whole amounts</td>
<td></td>
</tr>
<tr>
<td>Fractions of a set of objects (1)</td>
<td></td>
</tr>
<tr>
<td>Fractions of a set of objects (2)</td>
<td></td>
</tr>
<tr>
<td>Calculate fractions of a quantity</td>
<td></td>
</tr>
<tr>
<td>Problem solving – calculate quantities</td>
<td></td>
</tr>
</tbody>
</table>
Unit and Non-unit Fractions

Notes and Guidance

Children recap their understanding of unit and non-unit fractions from Year 2. They explain the similarities and differences between unit and non-unit fractions.

Children are introduced to fractions with denominators other than 2, 3 and 4, which they used in Year 2. Ensure children understand what the numerator and denominator represent.

Mathematical Talk

What is a unit fraction?
What is a non-unit fraction?
Show me $\frac{1}{2}$, $\frac{1}{3}$, $\frac{1}{4}$, $\frac{1}{5}$. What’s the same? What’s different?
What fraction is shaded? What fraction is not shaded?
What is the same about the fractions? What is different?

Varied Fluency

Complete the sentences to describe the images.

___ out of ___ equal parts are shaded.

of the shape is shaded.

Shade $\frac{1}{5}$ of the circle. Shade $\frac{3}{5}$ of the circle.

Circle $\frac{1}{5}$ of the beanbags. Circle $\frac{3}{5}$ of the beanbags.

What’s the same and what’s different about $\frac{1}{5}$ and $\frac{3}{5}$?

Complete the sentences.

A unit fraction always has a numerator of ____.
A non-unit fraction has a numerator that is _____ than ____.
An example of a unit fraction is ____.
An example of a non-unit fraction is ____.

Can you draw a unit fraction and a non-unit fraction with the same denominator?
True or False?

1/3 of the shape is shaded.

Sort the fractions into the table.

<table>
<thead>
<tr>
<th>Unit fractions</th>
<th>Fractions equal to one whole</th>
</tr>
</thead>
<tbody>
<tr>
<td>Non-unit fractions</td>
<td>Fractions less than one whole</td>
</tr>
<tr>
<td>3/4</td>
<td>3/5</td>
</tr>
</tbody>
</table>

Top left: Empty
Top right: 1/3, 1/4 and 1/2
Bottom left: 2/2 and 4/4
Bottom right: 3/4, 3/5, and 2/5
There are no unit fractions that are equal to one whole other than 1/1 but this isn’t in our list.

False, one quarter is shaded. Ensure when counting the parts of the whole that children also count the shaded part.
What is a Fraction?

Notes and Guidance

Children explore fractions in different representations, for example, fractions of shapes, quantities and fractions on a number line.

They explore and recap the meaning of numerator and denominator, non-unit and unit fractions.

Mathematical Talk

How can we sort the fraction cards? What fraction does each one represent? Could some cards represent more than one fraction? Is \( \frac{15}{3} \) an example of a non-unit fraction? Why?

Using Cuisenaire, how many white rods are equal to an orange rod? How does this help us work out what fraction the white rod represents?

Varied Fluency

Here are 9 cards.
Sort the cards into different groups. Can you explain how you made your decision? Can you sort the cards in a different way? Can you explain how your partner has sorted the cards?

Complete the Frayer model to describe a unit fraction.

Can you use the model to describe the following terms?

Use Cuisenaire rods.
If the orange rod is one whole, what fraction is represented by:
- The white rod
- The yellow rod
- The brown rod
Choose a different rod to represent one whole. What do the other rods represent now?

What is a Fraction?

Year 4 | Spring Term | Week 5 to 8 – Number: Fractions

Definition | Characteristics
---|---
Examples | unit fraction
Non-example | Numerator
Non-unit fraction | Denominator

Use Cuisenaire rods.
If the orange rod is one whole, what fraction is represented by:
- The white rod
- The red rod
- The yellow rod
- The brown rod
Choose a different rod to represent one whole. What do the other rods represent now?
What is a Fraction?

Reasoning and Problem Solving

Always, Sometimes, Never?

Alex says,

If I split a shape into 4 parts, I have split it into quarters.

Explain your answer.

Sometimes

If the shape is not split equally, it will not be in quarters.

Which representations of \(\frac{4}{5}\) are incorrect?

Explain how you know.

The image of the dogs could represent \(\frac{2}{5}\) or \(\frac{3}{5}\).

The bar model is not divided into equal parts so this does not represent \(\frac{4}{5}\).
Notes and Guidance

Children explore what a tenth is. They recognise that tenths arise from dividing one whole into 10 equal parts.

Children represent tenths in different ways and use words and fractions to describe them. For example, one tenth and $\frac{1}{10}$

Mathematical Talk

How many tenths make the whole?

How many tenths are shaded?

How many more tenths do I need to make a whole?

When I am writing tenths, the ___________ is always 10

How are fractions linked to division?

Varied Fluency

If the frame represents 1 whole, what does each box represent?

Use counters to represent:

• One tenth
• Two tenths
• Three tenths
• One tenth less than eight tenths

Identify what fraction of each shape is shaded.
Give your answer in words and as a fraction.

e.g.

Three tenths $\frac{3}{10}$

Annie has 2 cakes. She wants to share them equally between 10 people. What fraction of the cakes will each person get?

There are ___ cakes.
They are shared equally between ___ people.
Each person has $\square$ of the cake.

___ $\div$ ___ = ___

What fraction would they get if Annie had 4 cakes?
Fill in the missing values. Explain how you got your answers.

Children could use practical equipment to explain why and how, and relate back to the counting stick.

Odd One Out

Which is the odd one out? Explain your answer.

The marbles are the odd one out because they represent 8 or eighths. All of the other images have a whole which has been split into ten equal parts.
Count in Tenths

Notes and Guidance

Children count up and down in tenths using different representations.

Children also explore what happens when counting past $\frac{10}{10}$.

They are not required to write mixed numbers, however, children may see the $\frac{11}{10}$ as $1\frac{1}{10}$ due to their understanding of 1 whole.

Mathematical Talk

Let’s count in tenths. What comes next? Explain how you know.

If I start at ___ tenths, what will be next?

When we get to $\frac{10}{10}$ what else can we say? What happens next?

Varied Fluency

The counting stick is worth 1 whole. Label each part of the counting stick. Can you count forwards and backwards along the counting stick?

Continue the pattern in the table.
- What comes between $\frac{4}{10}$ and $\frac{6}{10}$?
- What is one more than $\frac{10}{10}$?
- If I start at $\frac{8}{10}$ and count back $\frac{4}{10}$, where will I stop?

Complete the sequences.
Teddy is counting in tenths. Teddy thinks that after ten tenths you start counting in elevenths. He does not realise that ten tenths is the whole, and so the next number in the sequence after ten tenths is eleven tenths or one and one tenth.

Can you spot his mistake?

**True or False?**

- Five tenths is $\frac{2}{10}$ smaller than 7 tenths.
- Five tenths is $\frac{2}{10}$ larger than three tenths.

Do you agree? Explain why.

This is correct. Children could show it using pictures, ten frames, number lines etc. For example:
Notes and Guidance

Children begin by using Cuisenaire or number rods to investigate and record equivalent fractions. Children then move on to exploring equivalent fractions through bar models.

Children explore equivalent fractions in pairs and can start to spot patterns.

Mathematical Talk

If the ___ rod is worth 1, can you show me $\frac{1}{2}$? How about $\frac{1}{4}$?

Can you find other rods that are the same? What fraction would they represent?

How can you fold a strip of paper into equal parts?
What do you notice about the numerators and denominators?
Do you see any patterns?

Can a fraction have more than one equivalent fraction?

Variied Fluency

The pink Cuisenaire rod is worth 1 whole.

Which rod would be worth $\frac{1}{4}$?

Which rods would be worth $\frac{2}{4}$?

Which rod would be worth $\frac{1}{2}$?

Use Cuisenaire to find rods to investigate other equivalent fractions.

Use two strips of equal sized paper. Fold one strip into quarters and the other into eighths. Place the quarters on top of the eighths and lift up one quarter, how many eighths can you see? How many eighths are equivalent to one quarter? Which other equivalent fractions can you find?

Using squared paper, investigate equivalent fractions using equal parts. e.g. $\frac{1}{4} = \frac{2}{8}$

Start by drawing a bar 8 squares along. Label each square $\frac{1}{8}$

Underneath compare the same length bar split into four equal parts. What fraction is each part now?
Explain how the diagram shows both $\frac{2}{3}$ and $\frac{4}{6}$.

The diagram is divided into six equal parts and four out of the six are yellow. You can also see three columns and two columns are yellow.

Which is the odd one out? Explain why.

This is the odd one out because the other fractions are all equivalent to $\frac{1}{2}$.

Teddy makes this fraction:

Mo says he can make an equivalent fraction with a denominator of 9.

Dora disagrees. She says it can't have a denominator of 9 because the denominator would need to be double 3.

Who is correct? Who is incorrect?

Mo is correct. He could make three ninths which is equivalent to one third.

Dora is incorrect. She has a misconception that you can only double to find equivalent fractions.
Children use Cuisenaire rods and paper strips alongside number lines to deepen their understanding of equivalent fractions. Encourage children to focus on how the number line can be divided into different amounts of equal parts and how this helps to find equivalent fractions e.g. a number line divided into twelfths can also represent halves, thirds, quarters and sixths.

**Mathematical Talk**

The number line represents 1 whole, where can we see the fraction \( \frac{1}{2} \)? Can we see any equivalent fractions?

Look at the number line divided into twelfths. Which unit fractions can you place on the number line as equivalent fractions? e.g. \( \frac{1}{2}, \frac{1}{3}, \frac{1}{4}, \frac{1}{5} \) etc. Which unit fractions are not equivalent to twelfths?

**Varied Fluency**

Use the models on the number line to identify the missing fractions. Which fractions are equivalent?

Complete the missing equivalent fractions.

Place these equivalent fractions on the number line.

Are there any other equivalent fractions you can identify on the number line?
Alex and Tommy are using number lines to explore equivalent fractions.

Alex is correct. Tommy’s top number line isn’t split into equal parts which means he cannot find the correct equivalent fraction.

Who do you agree with? Explain why.

Use the clues to work out which fraction is being described for each shape.

- My denominator is 6 and my numerator is half of my denominator.
  - I am equivalent to \( \frac{4}{12} \)
  - I am equivalent to one whole
  - I am equivalent to \( \frac{2}{3} \)

Can you write what fraction each shape is worth? Can you record an equivalent fraction for each one?

- Circle
- Triangle
- Square
- Pentagon

Accept other correct equivalences
Children use strip diagrams to investigate and record equivalent fractions.

They start by comparing two fractions before moving on to finding more than one equivalent fraction on a fraction wall.

Use two strips of equal sized paper. Fold one strip into quarters and the other into eighths. Place the quarters on top of the eighths and lift up one quarter; how many eighths can you see? How many eighths are equivalent to one quarter? Which other equivalent fractions can you find?

Using squared paper, investigate equivalent fractions using equal parts e.g. \( \frac{2}{4} = \frac{1}{2} \)

Start by drawing a bar 8 squares long. Underneath, compare the same length bar split into four equal parts.

How many fractions that are equivalent to one half can you see on the fraction wall?

Can a fraction have more than one equivalent fraction?

Can you use Cuisenaire rods or pattern blocks to investigate equivalent fractions?

Draw extra rows to show other equivalent fractions.
Reasoning and Problem Solving

How many equivalent fractions can you see in this picture?

Children can give a variety of possibilities. Examples:

\[
\frac{1}{2} = \frac{6}{12} = \frac{3}{6}
\]

\[
\frac{1}{4} = \frac{3}{12}
\]

Ron has two strips of the same sized paper.
He folds the strips into different sized fractions.
He shades in three equal parts on one strip and six equal parts on the other strip.
The shaded areas are equal.

What fractions could he have folded his strips into?

Ron could have folded his strips into sixths and twelfths, quarters and eighths or any other fractions where one of the denominators is double the other.

Eva says,

I know that \(\frac{3}{4}\) is equivalent to \(\frac{3}{8}\) because the numerators are the same.

Is Eva correct? Explain why.

Eva is not correct. \(\frac{3}{4}\) is equivalent to \(\frac{6}{8}\)

When the numerators are the same, the larger the denominator, the smaller the fraction.
Children continue to understand equivalence through diagrams. They move onto using proportional reasoning to find equivalent fractions.

Attention should be drawn to the method of multiplying the numerators and denominators by the same number to ensure that fractions are equivalent.

Using the diagram, complete the equivalent fractions.

\[
\frac{1}{4} = \frac{6}{12} \quad \frac{2}{3} = \frac{12}{18} \quad \frac{5}{12} = \frac{24}{24}
\]

Using the diagram, complete the equivalent fractions.

\[
\frac{1}{3} = \frac{6}{18} = \frac{12}{24} = \frac{24}{24}
\]

Complete:

\[
\frac{1}{4} = \frac{2}{8} = \frac{3}{12} = \frac{4}{20} = \frac{5}{50} = \frac{25}{125}
\]
Tommy is finding equivalent fractions.

\[
\frac{3}{4} = \frac{5}{6} = \frac{7}{8} = \frac{9}{10}
\]

He says,

I did the same thing to the numerator and the denominator so my fractions are equivalent.

Do you agree with Tommy? Explain your answer.

Tommy is wrong. He has added two to the numerator and denominator each time. When you find equivalent fractions you either need to multiply or divide the numerator and denominator by the same number.

Use the digit cards to complete the equivalent fractions.

Possible answers:

\[
\frac{1}{2} = \frac{3}{6}, \quad \frac{1}{2} = \frac{4}{8},
\]

\[
\frac{1}{3} = \frac{2}{6}, \quad \frac{1}{4} = \frac{2}{8},
\]

\[
\frac{3}{4} = \frac{6}{8}, \quad \frac{2}{3} = \frac{4}{6}
\]

How many different ways can you find?
Mathematical Talk

How many ___ make a whole?

If I have _____ eighths, how many more do I need to make a whole?

What do you notice about the numerator and denominator when a fraction is equivalent to a whole?

Varied Fluency

- Complete the part-whole models and sentences.

  There are ____ quarters altogether.

  ____ quarters = ____ whole and ____ quarter.

  Write sentences to describe these part-whole models.

- Complete. You may use part-whole models to help you.

  \[
  \frac{10}{3} = \frac{9}{3} + \frac{1}{3} = 3 \frac{1}{3}
  \]

  \[
  \frac{6}{3} + \frac{2}{3} = \frac{2}{3}
  \]

  \[
  \frac{16}{8} + \frac{3}{8} = \frac{19}{8}
  \]
### Fractions Greater than 1

#### Reasoning and Problem Solving

3 friends share some pizzas. Each pizza is cut into 8 equal slices. Altogether, they eat 25 slices. How many whole pizzas do they eat?

<table>
<thead>
<tr>
<th>3 friends share some pizzas. Each pizza is cut into 8 equal slices. Altogether, they eat 25 slices. How many whole pizzas do they eat?</th>
<th>They eat 3 whole pizzas and 1 more slice.</th>
<th>Rosie says, $\frac{16}{4}$ is greater than $\frac{8}{2}$ because 16 is greater than 8</th>
</tr>
</thead>
<tbody>
<tr>
<td>$\frac{13}{5} = 10$ wholes and 3 fifths</td>
<td>$\frac{10}{5} = 2$ wholes</td>
<td></td>
</tr>
<tr>
<td>$\frac{13}{5} = 2$ wholes and 3 fifths</td>
<td>Do you agree? Explain why.</td>
<td></td>
</tr>
</tbody>
</table>

---

Spot the mistake.

I disagree with Rosie because both fractions are equivalent to 4.

Children may choose to build both fractions using cubes, or draw bar models.
Children explore fractions greater than one on a number line and start to make connections between improper and mixed numbers.

They use cubes and bar models to represent fractions greater than a whole. This will support children when adding and subtracting fractions greater than a whole.

**How many ____ make a whole?**

Can you write the missing fractions in more than one way?

Are the fractions ascending or descending?

---

**Varied Fluency**

- Complete the number line.

- Draw bar models to represent each fraction.

- Fill in the blanks using cubes or bar models to help you.

- Write the next two fractions in each sequence.
  
  a) \(\frac{12}{7}, \frac{11}{7}, \frac{10}{7}, \ldots, \ldots\)
  
  b) \(3 \frac{1}{3}, 3, 2 \frac{2}{3}, \ldots, \ldots\)
  
  c) \(\frac{4}{11}, \frac{6}{11}, \frac{8}{11}, \ldots, \ldots\)
  
  d) \(12 \frac{3}{5}, 13 \frac{1}{5}, 13 \frac{4}{5}, \ldots, \ldots\)
Here is a number sequence.

\[
\frac{5}{12}, \frac{7}{12}, \frac{10}{12}, \frac{14}{12}, \frac{19}{12}, \ldots
\]

Which fraction would come next?
Can you write the fraction in more than one way?

The fractions are increasing by one more twelfth each time. The next fraction would be \(\frac{25}{12}\).

Play the fraction game for four players.
Place the four fraction cards on the floor. Each player stands in front of a fraction. We are going to count up in tenths starting at 0. When you say a fraction, place your foot on your fraction.

2 children can make four tenths by stepping on one tenth and three tenths at the same time. Alternatively, one child can make four tenths by stepping on \(\frac{2}{10}\) with 2 feet. With one foot, they can count up to 11 tenths or one and one tenth. With two feet they can count up to 22 tenths.

Circle and correct the mistakes in the sequences.

\[
\frac{5}{12}, \frac{8}{12}, \frac{11}{12}, \frac{15}{12}, \frac{17}{12}, \frac{9}{10}, \frac{7}{10}, \frac{6}{10}, \frac{3}{10}, \frac{1}{10}
\]

How can we make 4 tenths?
What is the highest fraction we can count to?
How about if we used two feet?
Children use practical equipment and pictorial representations to add two or more fractions with the same denominator where the total is less than 1.

They understand that we only add the numerators and the denominators stay the same.

Using your paper circles, show me what $\frac{1}{4} + \frac{1}{4}$ is equal to. How many quarters in total do I have?

How many parts is the whole divided into? How many parts am I adding? What do you notice about the numerators? What do you notice about the denominators?

Take a paper circle. Fold your circle to split it into 4 equal parts. Colour one part red and two parts blue. Use your model to complete the sentences.

_____ quarter is red.
_____ quarters are blue.
_____ quarters are coloured in.

Show this as a number sentence. $\frac{1}{4} + \frac{1}{4} = \frac{1}{4}$

We can use this model to calculate $\frac{3}{8} + \frac{1}{8} = \frac{4}{8}$.

Draw your own models to calculate

$\frac{1}{5} + \frac{2}{5} = \frac{3}{5}$

$\frac{2}{7} + \frac{3}{7} + \frac{1}{7} = \frac{6}{7}$

$\frac{7}{10} + \frac{9}{10} = \frac{16}{10}$

Eva eats $\frac{5}{12}$ of a pizza and Annie eats $\frac{1}{12}$ of a pizza. What fraction of the pizza do they eat altogether?
Rosie and Whitney are solving:

\[ \frac{4}{7} + \frac{2}{7} \]

Rosie says, \( \frac{4}{7} + \frac{2}{7} = \frac{6}{7} \)

Whitney says, \( \frac{4}{7} + \frac{2}{7} = \frac{6}{14} \)

Who do you agree with? Explain why.

Rosie is correct. Whitney has made the mistake of also adding the denominators. Children could prove why Whitney is wrong using a bar model or strip diagram.

Mo and Teddy share these chocolates.

They both eat an odd number of chocolates.
Complete this number sentence to show what fraction of the chocolates they each could have eaten.

\[ \square + \square = \frac{12}{12} \]

Possible answers:

\( \frac{1}{12} + \frac{11}{12} \)
\( \frac{3}{12} + \frac{9}{12} \)
\( \frac{5}{12} + \frac{7}{12} \)
(In either order)
Add 2 or More Fractions

Notes and Guidance

Children use practical equipment and pictorial representations to add two or more fractions. Children record their answers as an improper fraction when the total is more than 1.

A common misconception is to add the denominators as well as the numerators. Use bar models to support children’s understanding of why this is incorrect.

Children can also explore adding fractions more efficiently by using known facts or number bonds to help them.

Mathematical Talk

How many equal parts is the whole split into? How many equal parts am I adding?

Which bar model do you prefer when adding fractions? Why?

Can you combine any pairs of fractions to make one whole when you are adding three fractions?

Varied Fluency

Take two identical strips of paper.
Fold your paper into quarters.
Can you use the strips to solve

\[
\frac{1}{4} + \frac{1}{4} \quad \frac{1}{4} + \frac{1}{4} + \frac{1}{4} \quad \frac{3}{4} + \frac{3}{4} \quad \boxed{\frac{4}{4}} + \boxed{\frac{4}{4}} = \frac{7}{4}
\]

What other fractions can you make and add?

Use the models to add the fractions:

Choose your preferred model to add:

\[
\frac{2}{7} + \frac{2}{7} = \\
\frac{3}{5} + \frac{4}{5} = \\
\frac{7}{9} + \frac{4}{9}
\]

Use the number line to add the fractions.

\[
\frac{4}{9} + \frac{4}{9} + \frac{8}{9} = \\
\frac{4}{9} + \frac{5}{9} + \frac{8}{9} = \\
\frac{1}{9} + \frac{11}{9} + 1 = \\
\boxed{\frac{9}{9}} + \frac{5}{9} + \frac{7}{9} = \frac{17}{9}
\]
Add 2 or More Fractions

Reasoning and Problem Solving

Alex is adding fractions.

\[
\frac{3}{9} + \frac{2}{9} = \frac{5}{18}
\]

Is she correct? Explain why.

Any combination of ninths where the numerators total 11.

How many different ways can you find to solve the calculation?

Any combination of ninths where the numerators total 11.

Mo and Teddy are solving:

\[
\frac{6}{13} + \frac{5}{13} + \frac{7}{13}
\]

Mo

The answer is 1 and \( \frac{5}{13} \)

Teddy

The answer is \( \frac{18}{13} \)

Who do you agree with? Explain why.

They are both correct.

Mo has added \( \frac{6}{13} + \frac{7}{13} \) to make 1 whole and then added \( \frac{5}{13} \).

Alex is incorrect. Alex has added the denominators as well as the numerators.
Subtract Fractions

Notes and Guidance

Children use practical equipment and pictorial representations to subtract fractions with the same denominator within one whole.

They understand that we only subtract the numerators and the denominators stay the same.

Mathematical Talk

What fraction is shown first? Then what happens? Now what is left? Can we represent this in a number story?

Which models show take away? Which models show finding the difference? What’s the same? What’s different? Can we represent these models in a number story?

Can you partition \( \frac{9}{11} \) in a different way?

Varied Fluency

Eva is eating a chocolate bar. Fill in the missing information.

Can you write a number story using ‘first’, ‘then’ and ‘now’ to describe your calculation?

Use the models to help you subtract the fractions.

Complete the part whole models. Use equipment if needed. Can you write fact families for each model?
Subtract Fractions

Reasoning and Problem Solving

Find the missing fractions:

\[
\frac{7}{7} - \frac{3}{7} = \frac{2}{7} + \boxed{\frac{2}{7}}
\]

\[
\frac{7}{9} - \frac{5}{9} = \frac{4}{9} - \frac{2}{9}
\]

How many fraction addition and subtractions can you make from this model?

Jack and Annie are solving \(\frac{4}{5} - \frac{2}{5}\)

Jack's method: 

Annie's method: 

They both say the answer is two fifths. Can you explain how they have found their answers?

Jack has taken two fifths away. Annie has found the difference between four fifths and two fifths.

There are lots of calculations children could record. Children may even record calculations where there are more than 2 fractions e.g. \(\frac{3}{9} + \frac{1}{9} + \frac{3}{9} = \frac{7}{9}\)

Children may possibly see the red representing one fraction and the white another also.
Subtract 2 Fractions

Notes and Guidance

Children use practical equipment and pictorial representations to subtract fractions with the same denominator.

Encourage children to explore subtraction as take away and as difference. Difference can be represented on a bar model by using a comparison model and making both fractions in the subtraction.

Mathematical Talk

Have you used take away or difference to subtract the eighths using the strips of paper? How are they the same? How are they different?

How can I find a missing number in a subtraction? Can you count on to find the difference?

Can I partition my fraction to help me subtract?

Varied Fluency

Use identical strips of paper and fold them into eighths. Use the strips to solve the calculations.

\[
\begin{align*}
\frac{8}{8} - \frac{3}{8} &= \frac{5}{8} \\
\frac{7}{8} - \frac{3}{8} &= \frac{4}{8} \\
\frac{16}{8} - \frac{9}{8} &= \frac{7}{8} \\
\frac{13}{8} - \frac{8}{8} &= \frac{5}{8}
\end{align*}
\]

Use the bar models to subtract the fractions.

\[
\begin{align*}
\frac{6}{7} - \frac{2}{7} &= \frac{4}{7} \\
\frac{11}{6} - \frac{6}{6} &= \frac{5}{6} \\
\frac{13}{5} - \frac{6}{5} &= \frac{7}{5}
\end{align*}
\]

Annie uses the number line to solve \(\frac{17}{11} - \frac{9}{11}\).

Use a number line to solve:

\[
\begin{align*}
\frac{16}{13} - \frac{9}{13} &= \frac{7}{13} \\
\frac{16}{9} - \frac{9}{9} &= \frac{7}{9} \\
\frac{16}{7} - \frac{9}{7} &= \frac{7}{7} \\
\frac{16}{16} - \frac{9}{16} &= \frac{7}{16}
\end{align*}
\]
Match the number stories to the correct calculations.

Teddy eats \( \frac{7}{8} \) of a pizza. Dora eats \( \frac{5}{8} \).
How much do they eat altogether?

- Annie's model: \( \frac{7}{8} + \frac{5}{8} = \) 2
- Amir's model: \( \frac{7}{8} - \frac{5}{8} = \)

1st question matches with second calculation.
2nd question with first calculation.
3rd question with third calculation.

Teddy eats \( \frac{7}{8} \) of a pizza. Dora eats \( \frac{4}{8} \) less.
How much do they eat altogether?

- Annie's model: \( \frac{7}{8} + \frac{4}{8} = \) 1
- Amir's model: \( \frac{7}{8} - \frac{4}{8} = \)

How many different ways can you find to solve the calculation?

- Annie: \( \frac{7}{8} - \frac{3}{8} = \frac{4}{8} = \frac{1}{2} \)
- Amir: \( \frac{7}{8} - \frac{3}{8} = \frac{4}{8} = \frac{1}{2} \)

Children may give a range of answers as long as the calculation for the numerators is correct.

Annie and Amir are working out the answer to this problem.

\[ \frac{7}{9} - \frac{3}{9} \]

Annie uses this model.

Amir uses this model.

Which model is correct? Explain why.

Can you write a number story for each model?
Notes and Guidance

Children continue to use practical equipment and pictorial representations to subtract fractions.

Children subtract fractions from a whole amount. Children need to understand how many equal parts are equivalent to a whole e.g. \( \frac{9}{9} = 1, \frac{18}{9} = 2 \) etc.

Varied Fluency

- Use cubes, strips of paper or a bar model to solve:
  \[
  \frac{9}{9} - \frac{4}{9} = \frac{5}{9} \quad \frac{9}{9} - \frac{8}{9} = \frac{1}{9} \quad \frac{13}{9} - \frac{9}{9} = \frac{4}{9}
  \]

- What's the same? What's different?

- Jack uses a bar model to subtract fractions.
  \[
  2 - \frac{3}{4} = \frac{8}{4} - \frac{3}{4} = \frac{5}{4} = 1\frac{1}{4}
  \]

- Use Jack’s method to calculate.
  \[
  3 - \frac{3}{4} = \quad 3 - \frac{3}{8} = \quad 3 - \frac{7}{8} = \quad 3 - \frac{15}{8} =
  \]

- Dexter uses a number line to find the difference between 2 and \( \frac{6}{9} \).
  \[
  2 - \frac{6}{9} = 1\frac{3}{9}
  \]

Mathematical Talk

What do you notice about the numerator and denominator when a fraction is equal to one whole?

Using Jack’s method, what’s the same about your bar models? What’s different?

How many more thirds/quarters/ninths do you need to make one whole?
Dora is subtracting a fraction from a whole.

\[
5 - \frac{3}{7} = \frac{2}{7}
\]

Can you spot her mistake?

What should the answer be?

How many ways can you make the statement correct?

\[
2 - \frac{4}{8} = \frac{5}{8} + \frac{4}{8}
\]

Dora has not recognised that 5 is equivalent to \(\frac{35}{7}\).

\[
5 - \frac{3}{7} = \frac{33}{7} = 4\frac{5}{7}
\]

Lots of possible responses.  

\[
g.e. \\
2 - \frac{1}{8} = \frac{5}{8} + \frac{10}{8} \\
2 - \frac{7}{8} = \frac{5}{8} + \frac{4}{8} \\
2 - \frac{9}{8} = \frac{5}{8} + \frac{2}{8}
\]

Whitney has a piece of ribbon that is 3 metres long.

She cuts it into 12 equal pieces and gives Teddy 3 pieces.

How many metres of ribbon does Whitney have left?

\[
\frac{12}{4} - \frac{3}{4} = \frac{9}{4} = 2\frac{1}{4}
\]

Cutting 3 metres of ribbon into 12 pieces means each metre of ribbon will be in 4 equal pieces. Whitney will have \(2\frac{1}{4}\) to begin with.

Whitney has \(2\frac{1}{4}\) metres of ribbon left.
Fraction of an Amount (1)

Notes and Guidance

Children find a unit fraction of an amount by dividing an amount into equal groups.

They build on their understanding of division by using place value counters to find fractions of larger quantities including where they need to exchange tens for ones.

Mathematical Talk

Which operation do we use to find a fraction of an amount?

How many equal groups do we need?

Which part of the fraction tells us this?

How does the bar model help us?

Varied Fluency

Find $\frac{1}{5}$ of Eva’s marbles.

I have divided the marbles into equal groups.

There are marbles in each group.

$\frac{1}{5}$ of Eva’s marbles is marbles.

Dexter has used a bar model and counters to find $\frac{1}{4}$ of 12

Use Dexter’s method to calculate:

$\frac{1}{6}$ of 12  $\frac{1}{3}$ of 12  $\frac{1}{3}$ of 18  $\frac{1}{9}$ of 18

Amir uses a bar model and place value counters to find one quarter of 84

Use Amir’s method to find:

$\frac{1}{3}$ of 36  $\frac{1}{3}$ of 45  $\frac{1}{5}$ of 65
Whitney has 12 chocolates.

On Friday, she ate \( \frac{1}{4} \) of her chocolates and gave one to her mum.

On Saturday, she ate \( \frac{1}{2} \) of her remaining chocolates, and gave one to her brother.

On Sunday, she ate \( \frac{1}{3} \) of her remaining chocolates.

How many chocolates does Whitney have left?

Whitney has two chocolates left.

Fill in the Blanks

\[
\frac{1}{3} \text{ of } 60 = \frac{1}{4} \text{ of } \square
\]

\[
\frac{1}{5} \text{ of } 25 = \frac{1}{5} \text{ of } 25
\]
Children need to understand that the denominator of the fraction tells us how many equal parts the whole will be divided into. E.g. $\frac{1}{3}$ means dividing the whole into 3 equal parts.

They need to understand that the numerator tells them how many parts of the whole there are. E.g. $\frac{2}{3}$ means dividing the whole into 3 equal parts, then counting the amount in 2 of these parts.

What does the denominator tell us?
What does the numerator tell us?
What is the same and what is different about two thirds and two fifths?
How many parts is the whole divided into and why?
This is $\frac{3}{4}$ of a set of beanbags.

How many were in the whole set?

- Ron has £28

  On Friday, he spent $\frac{1}{4}$ of his money.

  On Saturday, he spent $\frac{2}{3}$ of his remaining money and gave £2 to his sister.

  On Sunday, he spent $\frac{1}{5}$ of his remaining money.

  How much money does Ron have left?

  What fraction of his original amount is this?

- Ron has £4 left.

  This is $\frac{1}{7}$ of his original amount.
Children use their knowledge of finding unit fractions of a quantity, to find non-unit fractions of a quantity. They use concrete and pictorial representations to support their understanding. Children link bar modelling to the abstract method in order to understand why the method works.

Mo has 12 apples.
Use counters to represent his apples and find:

- $\frac{1}{2}$ of 12
- $\frac{1}{4}$ of 12
- $\frac{1}{3}$ of 12
- $\frac{1}{6}$ of 12

Now calculate:

- $\frac{2}{2}$ of 12
- $\frac{3}{4}$ of 12
- $\frac{2}{3}$ of 12
- $\frac{5}{6}$ of 12

What do you notice? What’s the same and what’s different?

Use a bar model to help you represent and find:

- $\frac{1}{7}$ of 56 = 56 ÷ $\square$
- $\frac{2}{7}$ of 56
- $\frac{3}{7}$ of 56
- $\frac{4}{7}$ of 56
- $\frac{4}{7}$ of 28
- $\frac{7}{7}$ of 28

Whitney eats $\frac{3}{8}$ of 240 g bar of chocolate.
How many grams does she have left? Can you represent this on a bar model?
Reasoning and Problem Solving

Fractions of a Quantity

True or False?

To find $\frac{3}{8}$ of a number, divide by 3 and multiply by 8
Convince me.

False. Divide the whole by 8 to find one eighth and then multiply by three to find three eighths of a number.

Ron gives $\frac{2}{9}$ of a bag of 54 marbles to Alex.

Teddy gives $\frac{3}{4}$ of a bag of marbles to Alex.

Ron gives Alex more marbles than Teddy.

How many marbles could Teddy have to begin with?

Teddy could have 16, 12, 8 or 4 marbles to begin with.

$\frac{2}{9}$ of 54 $>$ $\frac{3}{4}$ of __
Children solve more complex problems for fractions of a quantity. They continue to use practical equipment and pictorial representations to help them see the relationships between the fraction and the whole.

Encourage children to use the bar model to solve word problems and represent the formal method.

If I know one quarter of a number, how can I find three quarters of a number?

If I know one of the equal parts, how can I find the whole?

How can a bar model support my working?

### Calculate Quantities

- **There are ____ counters in one part.**
  - 7 counters in one part.
- **There are ___ counters in one part.**
  - 4 counters in one part.
- **The whole is ____ of ____ = ____**
  - 5 of 24 = ___
- **The whole is ____ of ____ = ____**
  - 2 of ____ = ___
- **The whole is ____ of ____ = ____**
  - 3 of ____ = ___

### Varied Fluency

Use the counters and bar models to calculate the whole:

- **There are ___ counters in one part.**
- 3 counters in one part.
- **There are 7 counters in one part.**

### Complete.

- **The whole is 24**
  - \( \frac{1}{6} \) of 24 = __
  - \( \frac{5}{6} \) of 24 = __
- **The whole is ____**
  - \( \frac{2}{3} \) of ____ = ___
  - \( \frac{1}{5} \) of ____ = ___
  - \( \frac{3}{5} \) of ____ = ___

Jack has a bottle of lemonade.
He has one-fifth left in the bottle.
There are 150 ml left.
How much lemonade was in the bottle when it was full?
Reasoning and Problem Solving

The school kitchen needs to buy carrots for lunch. A large bag has 200 carrots and a medium bag has \( \frac{3}{5} \) of a large bag. Mrs Rose says,

Is Mrs Rose correct? Explain your reasoning.

Mrs Rose is correct. \( \frac{3}{5} \) of 200 = 120 Mrs Rose will need a large bag.

These three squares are \( \frac{1}{4} \) of a whole shape.

How many different shapes can you draw that could be the complete shape?

If \( \frac{1}{8} \) of \( A \) = 12, find the value of \( A \), \( B \), and \( C \).

\[
\frac{5}{8} \text{ of } A = \frac{3}{4} \text{ of } B = \frac{1}{6} \text{ of } C
\]

Lots of different possibilities. The shape should have 12 squares in total.

\[
A = 96 \\
B = 80 \\
C = 360
\]
Overview
Small Steps

- Recognise tenths and hundredths
- Tenths as decimals
- Tenths on a place value grid
- Tenths on a number line
- Divide 1-digit by 10
- Divide 2-digits by 10
- Hundredths
- Hundredths as decimals
- Hundredths on a place value grid
- Divide 1 or 2-digits by 100

Notes for 2020/21
This is new learning so there are no recap steps here. Children will need to explore the link with fractions and decimals using concrete manipulatives and pictorial representations. Using counters on a place value chart will help children see the connections when dividing by 10 and by 100.
Children recognise tenths and hundredths using a hundred square.

When first introducing tenths and hundredths, concrete manipulatives such as Base 10 can be used to support children’s understanding.

They see that ten hundredths are equivalent to one tenth and can use a part-whole model to partition a fraction into tenths and hundredths.

If each row is one row out of ten equal rows, what fraction does this represent?

If each square is one square out of one hundred equal squares, what fraction does this represent?

How many squares are in one row? How many squares are in one column? How many hundredths are in one tenth?

How else could you partition these numbers?

If the hundred square represents one whole:

Each square is ___ out of ___ equal squares.
Each square represents ___.
Each row is ___ out of ___ equal rows.
Each row represents ___.

Complete the table.

<table>
<thead>
<tr>
<th>Shaded</th>
<th>Tenths</th>
<th>Hundredths</th>
</tr>
</thead>
<tbody>
<tr>
<td>20 squares</td>
<td>$\frac{2}{10}$</td>
<td>$\frac{20}{100}$</td>
</tr>
<tr>
<td>4 columns</td>
<td></td>
<td></td>
</tr>
<tr>
<td>3 rows</td>
<td></td>
<td>$\frac{7}{10}$</td>
</tr>
</tbody>
</table>

We can use a part-whole model to partition 56 hundredths into tenths and hundredths.

Partition into tenths and hundredths:

- 65 hundredths
- $\frac{31}{100}$
- 80 hundredths
Reasoning and Problem Solving

Who is correct?

Amir is correct.

\[
\frac{50}{100} \text{ is equivalent to } \frac{5}{10}
\]

This can be demonstrated with Base 10 or a hundred square.

Dora

5 hundredths is equivalent to 50 tenths.

Amir

50 hundredths is equivalent to 5 tenths.

Explain why.

Ron says he can partition tenths and hundredths in more than one way.

Children may partition 42 hundredths as:

- 4 tenths and 2 hundredths
- 3 tenths and 12 hundredths
- 2 tenths and 22 hundredths
- 1 tenth and 32 hundredths
- 0 tenths and 42 hundredths

Other methods of partitioning are possible.

Use Ron’s method to partition 42 hundredths in more than one way.
Tenths as Decimals

Using the hundred square and Base 10, children can recognise the relationship between \( \frac{1}{10} \) and 0.1. Children write tenths as decimals and as fractions. They write any number of tenths as a decimal and represent them using concrete and pictorial representations. Children understand that a tenth is a part of a whole split into 10 equal parts. In this small step children stay within one whole.

Mathematical Talk

What is a tenth?

How many different ways can we write a tenth?

When do we use tenths in real life?

Which representation do you think is clearest? Why?

How else could you represent the decimal/fraction?

Notes and Guidance

Children write tenths as decimals and as fractions. They write any number of tenths as a decimal and represent them using concrete and pictorial representations.

Varied Fluency

Complete the table.

<table>
<thead>
<tr>
<th>Image</th>
<th>Words</th>
<th>Fraction</th>
</tr>
</thead>
<tbody>
<tr>
<td><img src="image1.png" alt="Image" /></td>
<td>five tenths</td>
<td>0.9</td>
</tr>
</tbody>
</table>

What fractions and decimals are represented in these diagrams?

![Diagram](image2.png)

How could you represent these decimals?

![Diagram](image3.png)

0.4 0.8 0.2

What's the same? What's different?
Tenths as Decimals

Reasoning and Problem Solving

Who is correct?

Annie

1.2 is equivalent to 1 whole and 2 tenths.

Dexter

1.2 is equivalent to 12 tenths.

Both children are correct. 1 whole is equal to 10 tenths so 1.2 is equal to 12 tenths.

What is the same? What’s different?

Show me.

six tens  six tenths

Children use concrete and pictorial representations to show the difference.

Which ten frame is the odd one out?

Three of the ten frames represent 0.5

This ten frame is the odd one out because it represents 6 tenths not 5 tenths.
Children read and represent tenths on a place value grid. They see that the tenths column is to the right of the decimal point.
Children use concrete representations to make tenths on a place value grid and write the number they have made as a decimal.
In this small step children will be introduced to decimals greater than 1.

How many ones are there?
How many tenths are there?
What's the same/different between 0.2, 1.2 and 0.8?
How many different ways can you make a whole using the three decimals?
Why do we need to use the decimal point?
How many tenths are equivalent to one whole?
Reasoning and Problem Solving

Use five counters and a place value grid. Place all five counters in either the ones or the tenths column.

How many different numbers can you make?

Describe the numbers you have made by completing the stem sentences.

There are __ ones and __ tenths.

___ ones + ___ tenths = ___

Two children are making eleven tenths.

Amir and Rosie have both made eleven tenths correctly. Amir has seen that 10 tenths is equivalent to 1 one.
Children read and represent tenths on a number line. They link the number line to measurement, looking at measuring in centimetres and millimetres.

Children use number lines to explore relative scale.

How many equal parts are between 0 and 1?
What are the intervals between each number?
How many tenths are in one whole?
What is 0.1 metres in millimetres?

Varied Fluency

Place the decimals on the number line.

Complete the number lines.

How long is the ribbon?

The ribbon is ___ metres long.
Reasoning and Problem Solving

What could the start and end numbers on the number line be? The start and end numbers could be 6 and 6.9 respectively, or 5.6 and 7.4. 

Explain your reasons. Children can find different start and end numbers by adjusting the increments that the number line is going up in.

Place the decimals on the number line.

Some children will draw on 20 intervals first. This method will allow them to identify where the numbers are placed but can be considered inefficient. Encourage children to think about the numbers first and consider which numbers are easiest to place e.g. 2.5 is probably easiest, followed by 1.9 or 2.9 etc.
Divide 1-digit by 10

Notes and Guidance

Children need to understand when dividing by 10 the number is being split into 10 equal parts and is 10 times smaller.

Children use counters on a place value chart to see how the digits move when dividing by 10. Children should make links between the understanding of dividing by 10 and this more efficient method.

Emphasise the importance of 0 as a place holder.

Mathematical Talk

What number is represented on the place value chart?

What links can you see between the 2 methods?

Which method is more efficient?

What is the same and what is different when dividing by 10 on a Gattegno chart compared to a place value chart?
Choose a digit card from 1 – 9 and place a counter over the top of that number on the Gattegno chart.

Ron says, To divide by 10, you need to move the counters to the right.

Do you agree? Use the Gattegno chart to explain your reason.

Ron is incorrect. Children will see that you move down one row to divide by 10 on a Gattegno chart whereas on a place value chart you move on column to the right.

Complete the number sentences.

\[ 4 \div 10 = 8 \div \square \div 10 \]
\[ 15 \div 3 \div 10 = \square \div 10 \]
\[ 64 \div \square \div 10 = 32 \div 4 \div 10 \]
As in the previous step, it is important for children to recognise the similarities and differences between the understanding of dividing by 10 and the more efficient method of moving digits. Children use a place value chart to see how 2 digit-numbers move when dividing by 10. They use counters to represent the digits before using actual digits within the place value chart.

What number is represented on the place value chart?

Do I need to use 0 as a place holder when dividing a 2-digit number by 10?

What is the same and what is different when dividing by 10 on a Gattegno chart compared to a place value chart?
Jack has used a Gattegno chart to divide a 2-digit number by 10. He has placed counters over the numbers in his answer.

Jack’s original number was 26. You can move each counter up one to multiply them by 10, which is the inverse to division.

What was Jack’s original number? How can you use the chart to help you?

Dexter says,

When I divide a 2-digit number by 10, my answer will always have digits in the ones and tenths columns.

Show that Dexter is incorrect.

Children should give an example of when Dexter is incorrect. For example, when you divide 80 by 10, the answer is 8 so there does not need to be anything in the tenths column.
Hundredths

Notes and Guidance

Children recognise that hundredths arise from dividing one whole into one hundred equal parts.

Linked to this, they see that one tenth is ten hundredths.

Children count in hundredths and represent tenths and hundredths on a place value grid and a number line.

Mathematical Talk

One hundredth is one whole split into how many equal parts?

How many hundredths can I exchange one tenth for?

How many hundredths are equivalent to 5 tenths? How does this help me complete the sequence?

How does Base 10 help you represent the difference between tenths and hundredths?

Varied Fluency

Complete the number lines.

\[
\begin{align*}
0 & \quad \frac{1}{100} & \quad \frac{2}{100} & \quad \frac{52}{100} & \quad \frac{54}{100} \\
\end{align*}
\]

Complete the sequences.

\[
\begin{align*}
\frac{27}{100} & , \frac{28}{100} & , \frac{31}{100} & \\
\frac{52}{100} & , \frac{51}{100} & , \frac{5}{10} & \\
\end{align*}
\]

Use fractions to complete the number lines.

\[
\begin{align*}
\frac{2}{10} & \quad \frac{3}{10} \\
\end{align*}
\]
Hundredths

Reasoning and Problem Solving

Here is a Rekenrek made from 100 beads.

If the Rekenrek represents one whole, what fractions have been made on the left and on the right?

On the left, there are 46 hundredths, this is equivalent to 4 tenths and 6 hundredths. On the right, there are 54 hundredths, this is equivalent to 5 tenths and 4 hundredths.

Children could also explore hundredths using a 100 bead string.

Complete the statements.

3 tenths and 2 hundredths = 2 tenths and __ hundredths

14 hundredths and 3 tenths = 4 tenths and __ hundredths

5 tenths and 1 hundredth < 5 tenths and __ hundredths

5 tenths and 1 hundredth > __ tenths and 5 hundredths

Can you list all the possibilities?

12
4
Anything more than 1
0, 1, 2, 3 or 4
Using the hundred square and Base 10, children can recognise the relationship between \( \frac{1}{100} \) and 0.01. Children write hundredths as decimals and as fractions. They write any number of hundredths as a decimal and represent the decimals using concrete and pictorial representations. Children understand that a hundredth is a part of a whole split into 100 equal parts. In this small step children stay within one whole.

**Mathematical Talk**

One hundredth is one whole split into \( \_ \_ \_ \) equal parts.

What is the same and what is different about a number written as a fraction and a number written as a decimal?

What is the same and different between 0.3 and 4 hundredths?

**Varied Fluency**

Complete the table.

<table>
<thead>
<tr>
<th>Image</th>
<th>Words</th>
<th>Fraction</th>
<th>Decimals</th>
</tr>
</thead>
<tbody>
<tr>
<td><img src="image1.png" alt="Image" /></td>
<td>56 hundredths</td>
<td></td>
<td></td>
</tr>
<tr>
<td><img src="image2.png" alt="Image" /></td>
<td></td>
<td>( \frac{17}{100} )</td>
<td></td>
</tr>
<tr>
<td><img src="image3.png" alt="Image" /></td>
<td></td>
<td></td>
<td>0.2</td>
</tr>
</tbody>
</table>

Write the number as a fraction and as a decimal.

How else could you represent this number?
Dora says,

17 hundredths is the same as 1,700

Is she correct? Explain your answer.

Dora is wrong as she has mistaken hundredths for hundreds.

Alex and Eva have been asked to write the decimal shaded on the 100 grid.

Alex says the grid shows 0.70

Eva says the grid shows 0.7

Who do you agree with? Explain your answer.

They are both correct. The grid shows 70 hundredths or 7 tenths and this is what Alex and Eva have given as their answers. In Alex's answer the 0 in the hundredths column isn't needed as it is not a place holder and doesn't change the value of the number.
Notes and Guidance

Children read and represent hundredths on a place value grid. They see that the hundredths column is to the right of the decimal point and the tenths column.

Children use concrete representations to make numbers with tenths and hundredths on a place value grid and write the number they have made as a decimal.

Mathematical Talk

What is a hundredth?

How many hundredths are equivalent to one tenth?

Look at the decimals you have represented on the place value grid and in the part whole models. What’s the same about the numbers? What’s different?

Varied Fluency

Write the decimal represented in each place value grid.

- There are ___ ones.
- There are ___ tenths.
- There are ___ hundredths.
- The decimal represented is ___

Make the decimals on a place value grid.

Use the sentence stems to describe each number.

Represent the decimals on a place value grid and in a part whole model. How many ways can you partition each number?

What is a hundredth?

How many hundredths are equivalent to one tenth?

Look at the decimals you have represented on the place value grid and in the part whole models. What’s the same about the numbers? What’s different?
Reasoning and Problem Solving

Use four counters and a place value grid. Place all four counters in either the ones, tenths or hundredths column.

How many different numbers can you make?

Describe the numbers you have made by completing the sentences.

There are __ ones, __ tenths and __ hundredths.

___ ones + ___ tenths + ___ hundredths = ___

Children can either make:
4, 3.1, 3.01, 2.2, 2.11, 2.02, 1.3, 1.21, 1.12, 1.03, 0.4, 0.31, 0.22, 0.13, 0.04

e.g. There are 2 ones, 0 tenths and 2 hundredths.

2 ones + 0 tenths + 2 hundredths = 2.02

Ron says he can partition 0.34 in more than one way.

Children may partition 0.45 into:
0 tenths and 45 hundredths
1 tenth and 35 hundredths
2 tenths and 25 hundredths
3 tenths and 15 hundredths
4 tenths and 5 hundredths

Other ways of partitioning are possible.

Use Ron’s method to partition 0.45 in more than one way.
Notes and Guidance

Children need to understand when dividing by 100 the number is being split into 100 equal parts and is 100 times smaller. Children use counters on a place value chart to see how the digits move when dividing by 100. Children should make links between the understanding of dividing by 100 and this more efficient method. Emphasise the importance of 0 as a place holder.

Mathematical Talk

What number is represented on the place value chart? Why is 0 important when dividing a one or two-digit number by 100? What is the same and what is different when dividing by 100 on a Gattegno chart compared to a place value chart? What happens to the value of each digit when you divide by 10 and 100?

Varied Fluency

Dexter uses counters to make a 1-digit number.

<table>
<thead>
<tr>
<th>Tens</th>
<th>Ones</th>
<th>Tenths</th>
<th>Hundredths</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
</tbody>
</table>

To divide the number by 100, we move the counters two columns to the right.

What is the value of the counters now?

Use this method to solve:

\[ 4 \div 100 = \_ \]
\[ 5 \div 100 = \_ \]
\[ \_ = 6 \div 100 \]

Here is a two-digit number on a place value chart.

<table>
<thead>
<tr>
<th>Tens</th>
<th>Ones</th>
<th>Tenths</th>
<th>Hundredths</th>
</tr>
</thead>
<tbody>
<tr>
<td>7</td>
<td>2</td>
<td></td>
<td></td>
</tr>
</tbody>
</table>

When dividing by 100, we move the digits 2 places to the _____.

\[ 72 \div 100 = \_ \]

Use this method to solve:

\[ 82 \div 100 = \_ \]
\[ \_ = 93 \div 100 \]
\[ 0.23 = \_ \div 100 \]
Divide 1 or 2-digits by 100

Reasoning and Problem Solving

Describe the pattern.

7,000 ÷ 100 = 70
700 ÷ 100 = 7
70 ÷ 100 = 0.7
7 ÷ 100 = 0.07

Can you complete the pattern starting with 5,300 divided by 100?

Children will describe the pattern they see e.g. 7,000 is 10 times bigger than 700, therefore the answer has to be 10 times bigger as the divisor has remained the same.

For 5,300:
5,300 ÷ 100 = 53
530 ÷ 100 = 5.3
53 ÷ 100 = 0.53
5.3 ÷ 100 = 0.053

Teddy says,

45 divided by 100 is 0.45 so I know 0.45 is 100 times smaller than 45

Mo says,

45 divided by 100 is 0.45 so I know 45 is 100 times bigger than 0.45

Who is correct? Explain your answer.

Teddy and Mo are both correct. Children may use a place value chart to help them explain their answer.